Honors Advanced Math

Name______________________

Review 1 - Answers

1. Look at the graph of each function to determine whether the function is even or odd. Even functions will have y-axis reflection symmetry. Odd functions will have origin symmetry (180 degree origin rotational symmetry).

a. $f(-x) = 3\tan(-x) + 2\sin(-x) = 3(-\tan x) + 2(-\sin(x)) = -3\tan x - 2\sin x$

$\therefore\ f(-x) = -f(x)$, so $f(x)$ is odd.

b. $f(-x) = \frac{2(-x)^2}{4-(-x)^2} = \frac{2x^2}{4-x^2}$.

$\therefore\ f(x) = f(-x)$, so f is even.

c. $f(-x) = (-x)^3 + 5 = -x^3 + 5$.

$\therefore\ f(x) \neq f(-x)$, $f(-x) \neq -f(x)$, so f is neither even nor odd.

2. Describe a sequence of graphical transformations to turn the graph of $f(x)$ into the graph of $g(x)$.

a. $f(x) = 2x^2 + 1, g(x) = -2(x+7)^2 + 5$

1. shift left by 7 units.
2. flip over x-axis.
3. shift up by 6 units.

b. either of these methods will work:

way 1: 1. shift left by 5. 2. shrink horizontally by a factor of 2 (half as wide).

way 2: 1. shrink horizontally by a factor of 2 (half as wide). 2. shift left by 5/2.

3. $(g \circ f)(x) = g(f(x)) = g(x^2 + 5) = \sqrt{x^2 + 5}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 5 = x + 5$

4. Suppose $g(x) = \frac{2x+3}{x-5}$. Find $\lim_{x\to\infty} g^{-1}(x)$.

One method: Calculate that $g^{-1}(x) = \frac{3+5x}{x-2}$, which has a horizontal asymptote at y = 5, so $\lim_{x\to\infty} g^{-1}(x) = 5$.

Another method: $g(x)$ has a vertical asymptote at $x = 5$, so $g^{-1}(x)$ has a horizontal asymptote at $y = 5$, so $\lim_{x\to\infty} g^{-1}(x)$

5. Find values of b and c that make $f(x)$ continuous on its domain.

Continuity at $x = -1$ requires that $1 - b + c = 10$; continuity at $x = 2$ requires that $4 + 2b + c = 1$. Solve the linear system (substitution, row reduction, inverse matrix) to get $b = -4$ and $c = 5$.

6. C must have 3 zeros (it is a cubic). Since $1-2i$ is a zero, $1+2i$ must also be a zero (real coefficients, complex conjugates theorem). C must have the form:

$C(x) = a(x-2)(x-(1-2i))(x-(1+2i))$, where a will be determined by the condition $f(1) = -12$.

$$C(x) = a(x^3 - 4x^2 + 9x - 10)$$

$$C(1) = a(1-4+9-10) = -12 \Rightarrow a = 3$$

7. $\frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{(ac+bd)+(cb-ad)i}{c^2+d^2} = (\frac{ac+db}{c^2+d^2}) + (\frac{cb-ad}{c^2+d^2})i$

the last expression can be though of as: (real number) + (real number)i.

8. **a.** $x^3 = x^2 + 23x + 42$

$x^3 - x^2 - 23x - 42 = 0$ possible rational zeros: $\left\{\frac{\text{facotrs of } 42}{\text{factors of } 1}\right\} = \{\pm 1,2,3,6,7,14,21,42\}$.

You can check each of the possible rational zeros using synthetic division, long division, or the remainder theorem. $x = 6$ is a rational zero. so factor out (x-6) to get:

$(x-6)(x^2 + 5x + 7) = 0$ use the quadratic formula to find the other two solutions:

solutions: $x = 6, \frac{-5 \pm \sqrt{3}i}{2}$.

b. $x^3 = -8$

same method can be used as in 8a. solutions are: $x = -2, 1 \pm \sqrt{3}i$. this problem can also be seen as "find the cube roots of -8." nth roots of complex numbers are covered in review 4.

9. Suppose $f(x) = \begin{cases} \cos x & \text{if} \quad x < \pi \\ \frac{2x^2 - 7x - 15}{x^2 - 9x + 20} & \text{if} \quad x \geq \pi \end{cases}$. Find

a. $\lim_{x \to \pi} f(x) =$ does not exist (left and right limits don't agree)

b. $\lim_{x \to \pi^-} f(x) = -1$

c. $\lim_{x \to 4^-} f(x) = -\infty$

d. $\lim_{x \to 5} f(x) = 13$ **e.** $\lim_{x \to -\infty} f(x) =$ does not exist (oscillates repeatedly between -1 and 1)

f. $\lim_{x \to \infty} f(x) = 2$ (horizontal asymptote)