Honors Advanced Math

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Review 3 Answers

1a. resulting vector should be <8,1/2>, graphically, 8 units right and 1/2 unit up.

b. $\cos\theta = \frac{v \cdot w}{|v| \cdot |w|} = \frac{-21}{15\sqrt{29}} \Rightarrow \theta \approx 105^\circ$

c. Twice the length of a unit vector in the dirction of **v**: $2 \cdot \frac{<12,-9>}{15} =< \frac{8}{5}, -\frac{6}{5} >$

d. $(-7,1) + \frac{2}{5} \cdot <12,-9> \; = \; (-\frac{11}{5}, -\frac{13}{5})$

2. a. $(x-2)^2 + (y+3)^2 = 1$

b. parametric: $\begin{cases} x(t) = 2 + 2\cos t \\ y(t) = -3 + 3\sin t \end{cases}$ Cartesian: $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

3. the vector equation of the line would be: $(x, y) = (0,2) + <3,-2> t$. This yields paramtric equations $\begin{cases} x(t) = 0 + 3t \\ y(t) = 2 - 2t \end{cases}$. This will be a ray on the interval: $0 \le t < \infty$.

4. a. $z = 1 + \mathrm{i} = \sqrt{2}cis45^\circ$. $z^8 = (\sqrt{2}cis45^\circ)^8 = 16cis360 = 16$. $|z| = \left|\sqrt{2}cis45^\circ\right| = \sqrt{2}$.

b. one of the 4th roots: $\sqrt[4]{\sqrt{2}cis(\pi/4)} = \sqrt[8]{2}cis(\pi/16)$ the other 3 are equally spaced around a circle of radius $\sqrt[8]{2}$. The other 3 would be: $\sqrt[8]{2}cis(\pi/16 + \pi/2)$, $\sqrt[8]{2}cis(\pi/16 + \pi)$, $\sqrt[8]{2}cis(\pi/16 + 3\pi/2)$.

5. This is equivalent to the equation $x^5 = 32$, which is equivalent to finding the five 5th roots of 32. one of the 5th roots of 32 is 2. the other 4 are equally spaced around a circle of radius 2. Concisely, the 5 solutions are: $2cis(\frac{360}{5}n)$, where n = 0,1,2,3,4.

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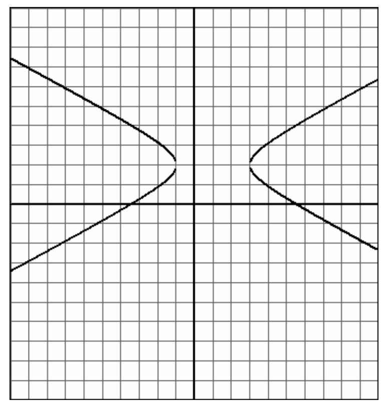
6. Fill in the conversion table below:

cartesian	Convert	polar
$(-\frac{1}{2},\frac{\sqrt{3}}{2})$	→	$(1,\frac{2\pi}{3})$
$(4\cos(\frac{5\pi}{6}),4\sin(\frac{5\pi}{6}))$	←	$(4,\frac{5\pi}{6})$
$x^2+y^2=12$	→	$r=\sqrt{12}$
$y=4$	←	$r=4\csc\theta$

7.

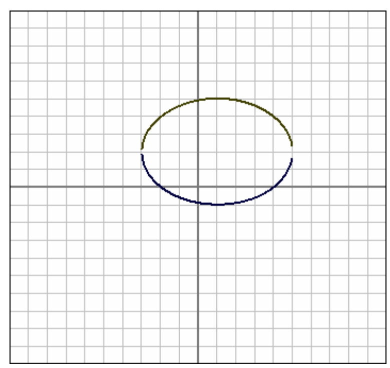
a. $x^2-4y^2-2x+16y-19=0$



$(0)^2-4(1)(-4)=16 \Rightarrow$ hyperbola

$\frac{(x-1)^2}{4}-\frac{(y-2)^2}{1}=1$

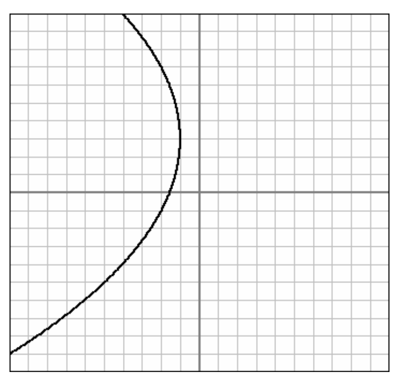
b. $9x^2+16y^2-18x-64y-71=0$



$(0)^2-4(9)(16)=(-) \Rightarrow$ ellipse

$\frac{(x-1)^2}{16}+\frac{(y-2)^2}{9}=1$

c. $y^2-6y+16x+25=0$



$(0)^2-4(1)(0)=0 \Rightarrow$ parabola

$(y-3)^2=-16(x+1)$

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8a. Vector: $(x,y,z)=\langle 2,-1,-3\rangle t+(-7,2,-1)$. Parametric: $x=2t-7,\ y=-t+2,$ $z=-3t-1$. Other answers are possible.

b. $3x-2y+z=d \to 3(4)-2(8)-1=-5 \to 3x-2y+z=-5$

c. $3(2t-7)-2(-t+2)+-3t-1=-5 \to t=\dfrac{21}{5} \to \left(\dfrac{7}{5},-\dfrac{11}{5},-\dfrac{68}{5}\right)$

9a. $\overrightarrow{AB} =< 6,-2,5 >$, $\overrightarrow{AC} =< 7,1,7 >$, $\left|\overrightarrow{AB}\right|=\sqrt{65}$, $\left|\overrightarrow{AC}\right|=\sqrt{99}$. Using the dot product we get: $\overrightarrow{AB}\cdot\overrightarrow{AC}=(6)(7)+(-2)(1)+(5)(7)=\sqrt{65}\sqrt{99}\cos\theta$, which yields $\theta\approx 20.78^\circ$.

b. $AREA=\dfrac{1}{2}\sqrt{65}\sqrt{99}\sin 20.78^\circ = 14.23$.

c. $(x, y, z) = A + s\,\overrightarrow{AB} + t\,\overrightarrow{AC}$

$(x, y, z) = (-1, 3, -2) + s <6, -2, 5> + t <7, 1, 7>$.

10.

$l_1 : (x,y,z)=\langle -2,-4,1\rangle t+(1,6,5)$

$l_2 : (x,y,z)=\langle 1,0,-5\rangle s+(-2,-2,2)$

a. Equate the x, y, and z coordinates to get this system:

$-2t + 1 = s - 2$
$-4t + 6 = -2$
$t + 5 = -5s + 2$

The system's solution is $t = 2$, $s = -1$, giving intersection point $(x, y, z) = (-3, -2, 7)$.

b. Let θ denote the angle between the lines' direction vectors.

$$\cos\theta=\frac{< -2,-4,1 > \cdot < 1,0,-5 >}{|< -2,-4,1 >|\ |< 1,0,-5 >|}=\frac{-7}{\sqrt{21}\sqrt{26}};\ \theta=\cos^{-1}\left(\frac{-7}{\sqrt{21}\sqrt{26}}\right)\approx 1.875 \text{ radians..}$$

It turns out that θ is the obtuse angle between the lines, so the acute angle is its supplement: $\pi - 1.875 \approx 1.267$ radians.

c. First get a vector perpendicular to the plane by calculating the cross product of the lines' direction vectors:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & 1 \\ 1 & 0 & -5 \end{vmatrix} = 20\mathbf{i}-9\mathbf{j}+4\mathbf{k}.$$

The plane's equation must be of the form $20x - 9y + 4z = D$. Substitute any point on the plane (such as the intersection found in part **a**) to get $D = -14$.

Answer: $20x - 9y + 4z = -14$.

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$P_1: 2x+2y+z=-2$

11. $P_2: 3x+2z=-2$

$P_3: -x+5y-2z=1$

a. $\left[\begin{array}{ccc|c} 2 & 2 & 1 & -2 \\ 3 & 0 & 2 & -2 \\ -1 & 5 & -2 & 1 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{array}\right]$ so the intersection is point (2, –1, –4).

b. $\left[\begin{array}{ccc|c} 2 & 2 & 1 & -2 \\ 3 & 0 & 2 & -2 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{1}{6} & -\frac{1}{3} \end{array}\right]$.

The reduced matrix represents the system $x+\frac{2}{3}z=-\frac{2}{3}$, $y-\frac{1}{6}z=-\frac{1}{3}$.

Let $z = t$ and solve the equations for x and y: $x=-\frac{2}{3}t-\frac{2}{3}$, $y=\frac{1}{6}t-\frac{1}{3}$.

Finally write the three parametric equations as a vector equation:

$(x, y, z) = <-\frac{2}{3}, \frac{1}{6}, 1> t + (-\frac{2}{3}, -\frac{1}{3}, 0)$.

Find the intersection of P_1 and P_2. If the intersection is a point, give the coordinates of the point. If the intersection is a line, give the equation of the line. If there is no intersection, explain how you know.

12. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Rewrite the matrix equation as a pair of transformation equations:

$x' = 3x + y$, $y' = 6x + 2y$.

Since $y' = 2x'$, the equation of the lline is $y = 2x$.