Honors Advanced Math

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Review 4 Answers

1. Let x = radium mass 5,000 years ago. Solve $x \cdot (\frac{1}{2})^{5000/1600} = 200$ to get $x \approx 1744.81$ mg.

2. $\log_3 450 = \log_3(2\cdot3\cdot3\cdot5\cdot5) = \log_3 2 + \log_3 3 + \log_3 3 + \log_3 5 + \log_3 5 = a + 2 + 2b$.

3. $g(x) = (\frac{1}{8})^x = (2^{-3})^x = ((4^{1/2})^{-3})^x = 4^{-\frac{3}{2}x} = f(-\frac{3}{2}x)$, so the transformations are:

- reflection across the y-axis
- horizontal shrink by a factor of 3/2

done in either order.

4. a. Domain is all real numbers x such that $x > 2$ or $x < -2$. Range is all real numbers.

b. $\lim_{x\to(-2)-} F(x) = -\infty$ and $\lim_{x\to 2+} F(x) = -\infty$.

5. Applying the change-of-base formula: $\log_b c = \frac{\log_c c}{\log_c b} = \frac{1}{\log_c b}$

6. a. $(1,0) \to (a, b)$; $(0,1) \to (c, d)$.

b. True: $x' = 0x + 0y = 0$ and $y' = 0x + 0y = 0$.

7. a. $x' = -y$, $y' = -x$.

b. $x' = (\cos\theta)x + (\sin\theta)y$, $y' = (-\sin\theta)x + (\cos\theta)y$.

8. a. $\begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1.598 \\ 3.232 \end{bmatrix}$.

b. $\begin{bmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$;

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{bmatrix}^{-1} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \approx \begin{bmatrix} 0.598 \\ -4.964 \end{bmatrix}$.

9. Substitute each of the 4 given points in place of x and y in the equation $y = ax^3 + bx^2 + cx + d$. This gives a system of four equations in four variables. Solve using rref to get a, b, c, and d. Answer: $f(x) = 0.25x^3 + 0x^2 - 1x + 3$.

10. Subtract $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ from both sides; multiply both sides by –1; left-multiply both sides by the inverse of the 2-by-2 matrix. Answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 13/8 \end{bmatrix}$.

11. Setup is $\frac{3x+4}{x^2+14x+48} = \frac{A}{x+6} + \frac{B}{x+8}$. Multiply both sides by $(x + 6)(x + 8)$ to get $3x + 4 = A(x + 8) + B(x + 6) = (A + B)x + (8A + 6B)$.

Equate coefficients to get the system $3 = A + B$, $4 = 8A + 6B$. Solution is $A = -7$, $B = 10$.

Answer: $\frac{3x+4}{x^2+14x+48} = \frac{-7}{x+6} + \frac{10}{x+8}$.