Part A. Multiple Choice

1.	B	**6.**	E
2.	D	**7.**	A
3.	B	**8.**	B
4.	D	**9.**	D
5.	E	**10.**	A

Part B. Written Response Questions

11. There are a couple of ways to do this. The longer of which would be to just do a synthetic division (or long division). The quicker way is to use the remainder theorem and plug –1 into the function. The result is –6.

12. This is just a log manipulation problem. Here's how it goes:

$$\log_7 (x+1)(x-5) = 1$$
$$7^1 = (x+1)(x-5)$$
$$0 = x^2 - 4x - 12$$
$$0 = (x-6)(x+2)$$
$$x = 6 \text{ or } x = -2$$

But, checking each of the solutions, we see that $x = -2$ is not a solution.

13. This could be done graphically, as long as you can find the exact answer and generalize to all solutions. The analytic approach is not too bad:

$$\sec^2 x - 1 - 4\sec x + 5 = 0 \text{ (Pythagorean identity)}$$
$$\sec^2 x - 4\sec x + 4 = 0$$
$$x^2 - 4x + 4 = 0 \text{ (Substitute for quad form}$$
$$(x-2)(x-2) = 0$$
$$x = 2$$
$$\sec x = 2 = \tfrac{1}{\cos x}$$
$$\cos x = 1/2$$
$$x = \pm\tfrac{\pi}{3} + 2\pi n$$

Be careful on problems like this not to leave out any solutions.

14. This is an infinite geometric series (Check the first couple of terms…). It will converge if the common ratio is less than one, so:

$|r| < 1$

$\left|\frac{-3}{x}\right| < 1$

$(x > 3) \cup (x < -3)$

The final result could have been found by solving the inequality analytically or graphing the inequality.

Part C. Written response

15. This is a standard find the area of the quadrilateral problem. I start by drawing a diagonal from the lower left-hand corner to the upper-right hand corner. I'll call this diagonal *d*, the triangle below I'll call region 1, and the triangle above I'll call region 2. My general procedure will be to get enough sides and angles for each of the two triangles to use one of our triangle area formulas. Then I'll add the area of the 2 triangles together.

By the law of cosines,

$$d^2 = 300^2 + 200^2 - 2(300)(200)\cos 115^\circ$$
$$d = 425.10\, ft$$

I'll now find an angle. I chose the angle in region 1 near the vertex with the 70 degree angle. I'll call this angle A.

By the law of sines,

$$\frac{\sin A}{200} = \frac{\sin 115^\circ}{425.10}$$
$$A = 25.24^\circ$$

This gives enough information to find the area of region 1:

$$Area1 = .5(300)(200)\sin 115 = 27{,}189.23$$

For region 2 we only need an angle. Subtract 70-25.24= 44.76. This gives an area of region 2:

$$Area2 = .5(250)(425.10)\sin 44.76^\circ = 37{,}417.37$$

Total Area = Area 1 + Area 2 = **64,606.60 square feet.**

16. a) <u>Graphical</u>: *f* must pass the Vertical Line Test (to determine whether it is a function) and the Horizontal Line Test (to determine whether it is one-to-one). <u>Non-graphical</u>: for every *x* in the domain of *f* there must exist exactly 1 *y* in the range of *f* (definition of a function) and vise-versa (one-to-one).

b) Since an even function is, by definition, symmetric over the y-axis it could never pass the Horizontal Line Test. Therefore, an even function can *never* have an inverse because it is not one-to-one.

c) Domain: $0 \le x < \infty$; Range: $1 \le y \le 5$

d) $$g(h(x)) = g(e^x) = \ln e^x = x$$
$$h(g(x)) = h(\ln x) = e^{\ln x} = x$$

Since $g(h(x))=h(g(x))$ we know that g and h are inverses.

17. Analyzing each piece of information, we know:

1.Degree is less than or equal to 4.
2.The graph passes through (0,3).
3.There is a zero at (1,0).
4.There is a double zero at (2,0).
5.The graph goes down on the right and up on the left.

We know the poly must look like (x-2)(x-2)(x-1). Because of 5, we know that the graph can not be a 4th degree polynomial, so these are all the zeroes. We now need to scale the graph to go through (0,3).

$$P(x) = a(x-1)(x-2)(x-2)$$
$$P(0) = a(-1)(-2)(-2) = 3,$$
$$a = -3/4$$

$$P(x) = -3/4(x-1)(x-2)^2$$

Just making a graph consistent with the above information gets to remainder of the points.

18. **a.** $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ and $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

b. rotation clockwise by π/3, centered at the origin

c. Solve $\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ using any method. (For example, you could use rref, or you could solve using an inverse matrix.) The answer is approximately (3.23, –1.60).

d. Since T is a rotation centered at the origin, it does not change any point's distance from the origin. Therefore, $|\, T\mathbf{v}\,| = |\,\mathbf{v}\,|$.