Name ____

Teacher (circle): Kelly Normile Verner

Class block (circle): A B C D E H

Honors Advanced Math
Final Exam 2007
Lexington High School
Mathematics Department

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.

The exam has 3 parts. Directions for each part appear below.

In total, there are 54 points that you can earn. A letter grade scale will be set by the course faculty after the tests have been graded.

Part A. Short Problems

7 questions, 2 points each, 14 points total

If your answer is correct, you will receive full credit. Showing work is not required.

If your answer is incorrect, you may receive half credit if you have shown some correct work.

A good pace on this part would be to spend 2-5 minutes per problem.

Part B. Medium Problems

6 problems, 4 points each, 24 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

A good pace on this part would be to spend 4-7 minutes per problem.

Part C. Long Problems

2 problems, 8 points each, 16 points total

Write a complete, clearly explained solution to each problem. Partial credit will be given.

A good pace on this part would be to spend 8-10 minutes per problem.

Part A. Short Problems

7 problems, 2 points each, 14 points total

1. Find all solutions to the equation $z^5 + 1 = 4 - 3i\sqrt{3}$. Give your answers in $r \text{ cis } \theta$ form.

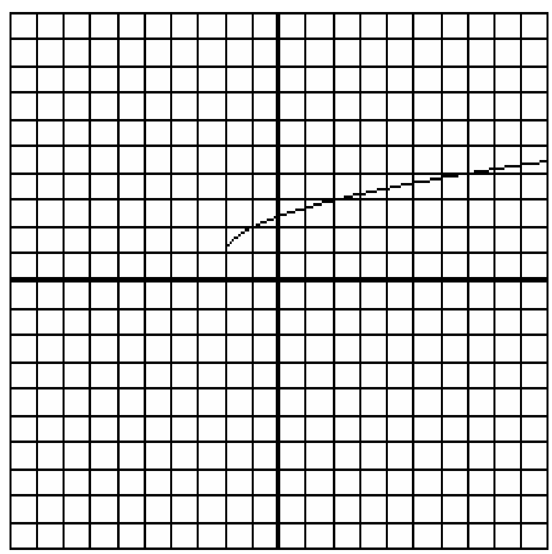
2. Find the equations of the asymptotes of the hyperbola $\frac{(y+3)^2}{4} - \frac{(x+7)^2}{25} = 1$.

3. Suppose **v** = <–3, –5> and **w** = <–7, 4>. Express the vector <21, –59> as a linear combination of **v** and **w**. In other words, find *a* and *b* so that <21, –59> = *a***v** + *b***w**.

4. Let $f(x)=\begin{cases} \sin x & \text{if } -\frac{\pi}{2}\le x\le\frac{\pi}{6} \\ \frac{x^2+2x-24}{x-4} & \text{if } \frac{\pi}{6}<x \end{cases}$. Find the domain and range of $f(x)$.

5. Let $f(x)$ be given by the graph at the right, and let $g(x)=5^{x-3}+7$. Find $(f^{-1}\circ g^{-1})(12)$.

y = f(x)



6. The equations of two planes are given below:

Plane 1: $x + y + z = 7$ Plane 2: $3x + y - 5z = 11$

Find the vector equation of the line that is formed where the two planes intersect.

7. Suppose $f(x)$ is an ***odd function***, and $f(4) = -5$.

Suppose T is a linear transformation of the plane whose matrix is $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$.

Let $g(x)$ stand for the function obtained by applying transformation T to the graph of $f(x)$.

What is the value of $g(-12)$?

Part B. Medium Problems

6 problems, 4 points each, 24 points total

1. Using an algebraic (non-graphical) method, ***without using your calculator***, find all solutions to the equation $\cos^2(x) + \cos(x) = \sin^2(x)$. Give exact answers in radians (not decimal approximations).

2. Suppose $p(x)$ is a 4th degree polynomial with the following properties:

- The coefficients of $p(x)$ are real numbers.
- The point $(4,0)$ is a local maximum.
- $p(2-i)=0$ and $p(1)=-54$.

Find a formula for $p(x)$. You may give your answer in factored form.

3. Prove that $\log(MN) = \log M + \log N$. You may assume the rules of exponents and the technique for switching equations between logarithmic and exponential forms. You may not assume any other properties of logarithms.

4. Consider the function $f(x) = \dfrac{x+3}{(x+7)(x-1)}$.

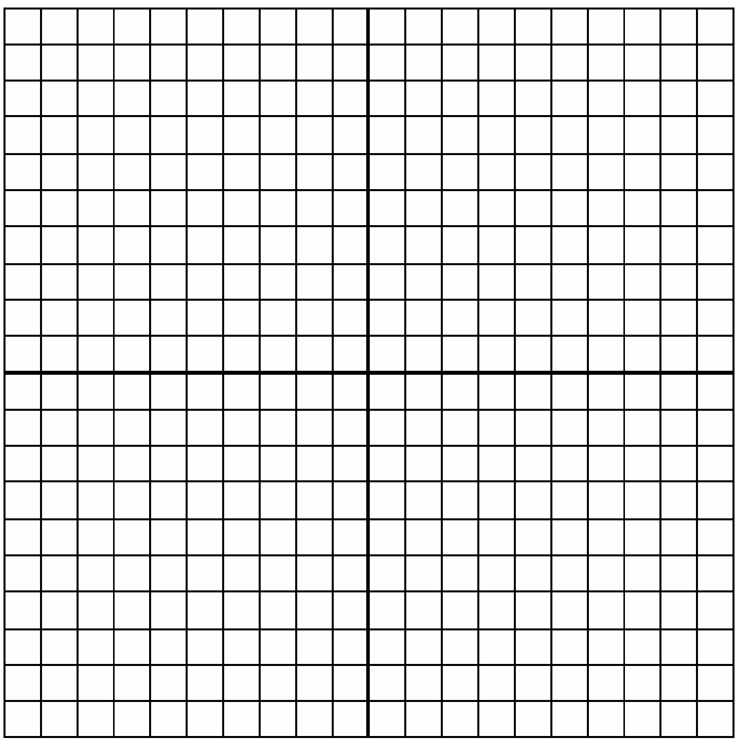
a. Give a partial fraction decomposition of $\dfrac{x+3}{(x+7)(x-1)}$. In other words, find A and B so that $\dfrac{x+3}{(x+7)(x-1)} = \dfrac{A}{x+7} + \dfrac{B}{x-1}$.

b. Let $g(x) = f(x-3)$. Prove that $g(x)$ is an odd function.

5. Function $f(x)$ has the following properties:

- The domain of $f(x)$ is $(-4,-2)\cup(-2,\infty)$ and the range of $f(x)$ is $(-5,-3]\cup(-1,1)\cup(1,\infty)$.
- $f(x)$ is continuous for all x in its domain except for $x = 1$ and $x = -2$.
- $\lim\limits_{x\to -4^+} f(x) = -1$, $\lim\limits_{x\to -2} f(x) = 1$, $\lim\limits_{x\to \infty} f(x) = -5$, and $\lim\limits_{x\to 1^-} f(x) = \infty$.
- $f(1) = -3$.

Sketch a possible graph of $f(x)$ on the graph below. Include the coordinates of any important points and the equations for any asymptotes. ***Note**: To get full credit on this problem, your graph must be **clear**. There should be no ambiguity about open or closed circles. If you make a mistake and erase something, be sure to erase completely.*



6. Suppose that the depth of the water in a harbor is 20 feet at low tide, 30 feet at high tide, and fluctuates in such a way that it can be modeled with a sinusoidal function.

Let t represent time measured in hours, and let $D(t)$ represent the water's depth in feet at time t. Suppose that a low tide occurs at time $t = 5.4$ and the next high tide occurs at time $t = 11.6$.

a. Using the sine function, write a function formula for $D(t)$.

b. Find all times t at which the depth of the water is 28 feet. (Use decimal approximations accurate to the nearest 0.01.)

Part C. Long Problems

2 problems, 8 points each, 16 points total

1. Suppose that an object moves along a line L_1, with its position (x, y, z) at time t given by $x = -8t + 7$, $y = -4t - 9$, and $z = 2t - 4$, for all real numbers t.

a. How fast is the object moving, per unit of time?

b. Find the coordinates of the point, P, where L_1 intersects the plane $-x + 4y - 3z = -3$.

c. Find the equation of the line L_2 going through P that is perpendicular to the plane $-x + 4y - 3z = -3$.

d. Find the measure of the angle formed at the intersection of L_1 and L_2. Give your answer in degrees, as a decimal approximation accurate to the nearest 0.01°.

2. Consider quadrilateral $WXYZ$ where $W = (-1, -4)$, $X = (4, -5)$, and $Y = (8, 2)$, and $Z = (-1, 4)$.

a. The diagonals of $WXYZ$ are segments WY and XZ. Calculate the vector dot product $\vec{WY} \cdot \vec{XZ}$, then determine whether or not the diagonals are perpendicular.

b. Find the radian measure of angle XYZ. Give a decimal approximation accurate to the nearest 0.01 radian.

c. Find the area of $WXYZ$. You may use a decimal approximation to the nearest 0.01.