

# Level 1 Advanced Math 2005 Final Exam

NAME: \_\_\_\_\_

## Instructions

WRITE ANSWERS IN THE SPACES PROVIDED AND SHOW ALL WORK. Partial credit will not be given if work is not shown. Ask for extra paper if you need it. NOTHING ON THE EXTRA PAPER will be graded unless you explicitly write "see extra paper" on this exam paper AND you clearly indicate the problem number on the extra paper. Make sure your name is on any paper you want graded.

CALCULATORS are permitted. FORMULA SHEETS are attached at the end of the exam. You may detach these if you wish.

ALL PAPERS (the exam, extra paper, and the formula sheets) will be collected at the end of the exam.

POINT VALUES are marked on each problem. 100 points total

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1. (5 points, 1 point per part) The definition of a piecewise function is given below. Answer the questions that follow.

$$f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ |x| - 5 & \text{if } -1 \leq x \leq 2 \\ x^2 & \text{if } 2 < x < 2\pi \end{cases}$$

Find:

a.  $f(0)$  \_\_\_\_\_      b.  $f(1.6)$  \_\_\_\_\_      c.  $f(-2.3)$  \_\_\_\_\_

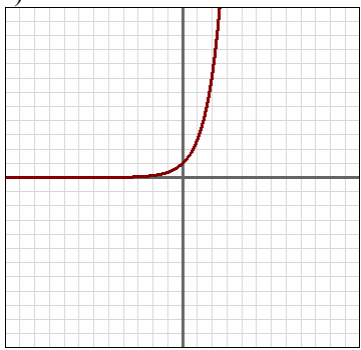
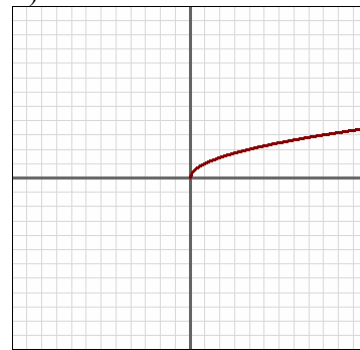
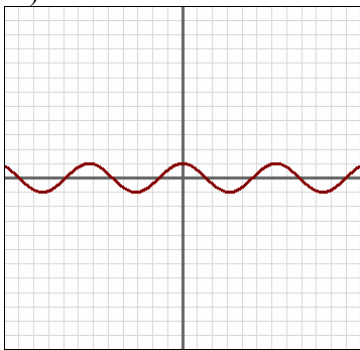
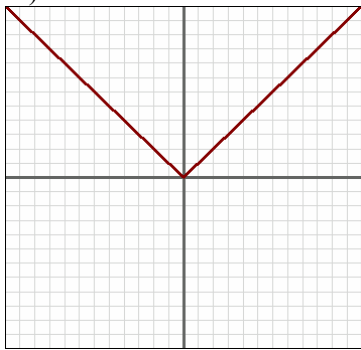
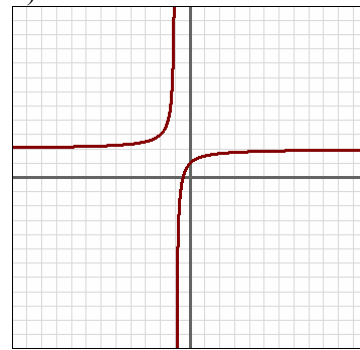
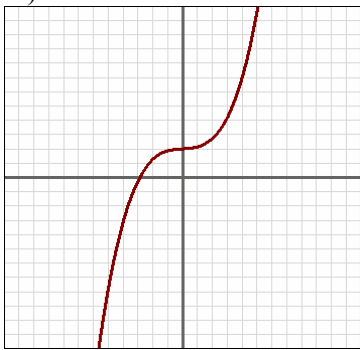
d.  $f(2)$  \_\_\_\_\_      e. The domain of  $f$ : \_\_\_\_\_

2. (2 points) State the domain of the function  $f(x) = \sqrt{x+1} + \frac{x+2}{x-2}$

Domain: \_\_\_\_\_

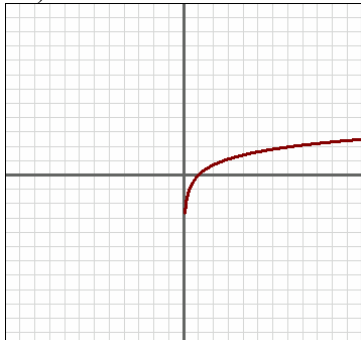
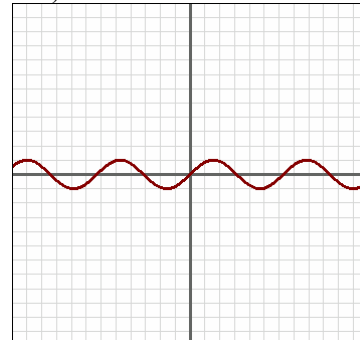
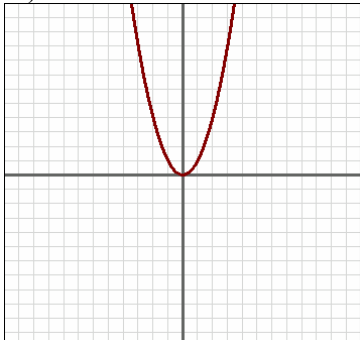
3. (9 points, 1 points for each of 9 parts) Below are 12 algebraic formulas and nine graphs of some of the functions we studied this year. For each graph, state which of the formulas (A—L) best matches the graph. You will not use all of the formulas. THE GRAPHS CONTINUE ON THE NEXT PAGE.

A) $f(x) = a(x - h)^2 + k$	B) $f(x) = A x - C  + D$	C) $f(x) = \frac{Ax + B}{Cx + D}$
D) $f(x) = A \sin Bx$	E) $f(x) = A \cos Bx$	F) $f(x) = A \tan Bx$
G) $f(x) = mx + b$	H) $f(x) = AB^{kx} + D$	I) $f(x) = A \log_B(x + C) + D$
J) $f(x) = A\sqrt{x + C} + D$	K) $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$	L) $f(x) = A[B(x + C)] + D$ $f(x) = A[B(x + C)] + D$ (These are alternate notations for the same function)

<p>i)</p>  <p>a) Fomula _____</p>	<p>ii)</p>  <p>a) Fomula _____</p>	<p>iii)</p>  <p>a) Fomula _____</p>
<p>iv)</p>  <p>a) Fomula _____</p>	<p>v)</p>  <p>a) Fomula _____</p>	<p>vi)</p>  <p>a) Fomula _____</p>

(Formulas repeated for reference)

A) $f(x) = a(x - h)^2 + k$	B) $f(x) = A x - C  + D$	C) $f(x) = \frac{Ax + B}{Cx + D}$
D) $f(x) = A \sin Bx$	E) $f(x) = A \cos Bx$	F) $f(x) = A \tan Bx$
G) $f(x) = mx + b$	H) $f(x) = AB^{kx} + D$	I) $f(x) = A \log_b(x + C) + D$
J) $f(x) = A\sqrt{(x + C)} + D$	K) $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$	L) $f(x) = A[B(x + C)] + D$ $f(x) = A[B(x + C)] + D$ (These are alternate notations for the same function)

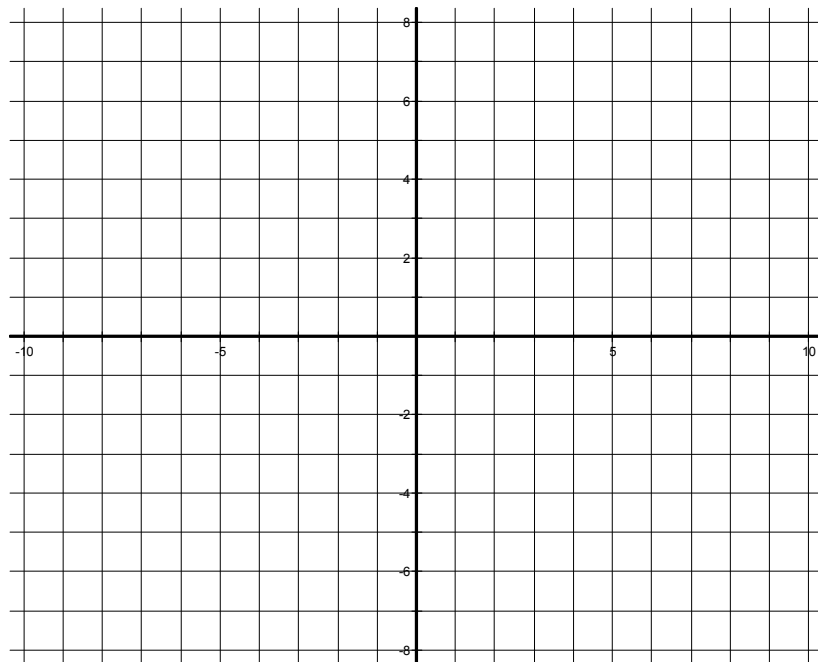
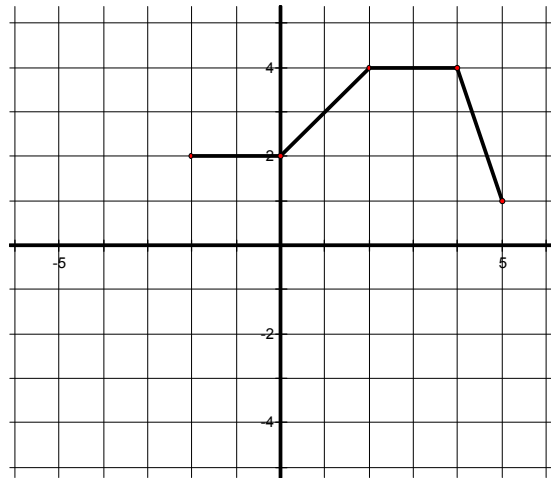
vii)	viii)	ix)
		
a) Formula _____	a) Formula _____	a) Formula _____

4. (4 points, 2 points per part) Let  $f(x) = \sqrt{3x + 7}$

a. Compute the inverse function  $f^{-1}(x)$  \_\_\_\_\_

b. Compute  $(f^{-1} \circ f)(x)$ : \_\_\_\_\_

5. (3 points) The graph of a function  $f(x)$  is given below. Sketch a graph of the transformation  $g(x) = -f\left(\frac{1}{2}x\right) - 2$  on the empty grid below.



6. (2 points) Let  $h(x) = \sqrt{3x^2 + 1}$ . Write  $h(x)$  as a composition of two functions.  
 $h(x) = (f \circ g)(x)$  THE IDENTITY FUNCTION  $f(x) = x$  MAY NOT BE PART OF YOUR ANSWER. (Hint: there is more than one correct answer).

$g(x) =$  \_\_\_\_\_

$f(x) =$  \_\_\_\_\_

7. (6 points) The functions  $f(x)$  and  $g(x)$  are defined in the first three columns of the table below. Complete the fourth and fifth columns. If a function is undefined for a given input, then write “undefined” in the table.

$x$	$f(x)$	$g(x)$	$(f \circ g)(x)$	$(f \div g)(x)$
0	3	2		
1	5	0		
2	1	3		

8. (3 points) Find the zeros of this function. You may give your answers in radical form (like  $5\sqrt{7}$ ) or decimal form to the nearest hundredth (like 13.23). If there are no zeros, write “no zeros”

$f(x) = x^4 - 5x^2 + 6$

Zeros: \_\_\_\_\_

9. (3 points) What is the remainder when  $x^4 + 3x^3 - 11x^2 - 3x + 10$  is divided by  $(x - 2)$ ?

Remainder: \_\_\_\_\_

10. (2 points) Which rational function below has the following features: asymptotes  $x = -2$  and  $y = 1$  and a hole at  $x = 3$  ?

a.  $y = \frac{(x-3)(x+3)}{(x+2)(x-3)}$

b.  $y = \frac{(x-3)}{(x+2)(x-3)}$

c.  $y = \frac{(x-3)(x+3)}{(x-2)(x-3)}$

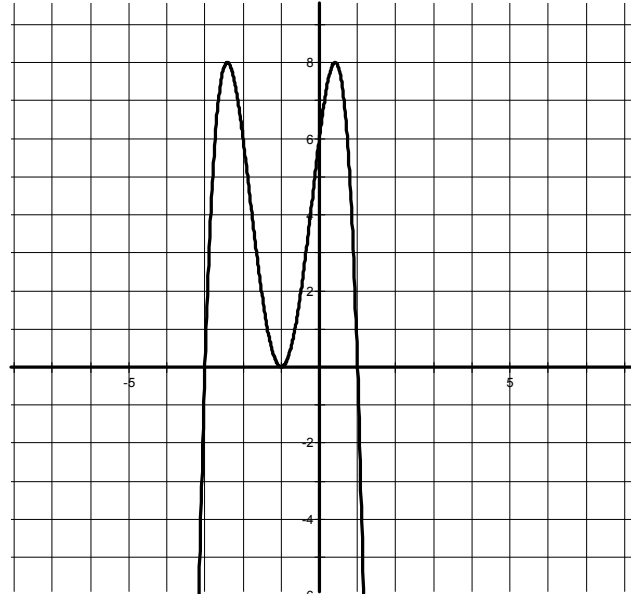
11. (2 points) Simplify:  $e^{2\ln x}$

Answer: \_\_\_\_\_

12. (3 points) Solve for x:  $\log x + \log(x+3) = 1$

$x =$  : \_\_\_\_\_

13. (8 points, as marked) The graph of a polynomial function is shown at right.



a. (1 point) Is the leading coefficient positive or negative?

\_\_\_\_\_

b. (1 point) Is the degree even or odd?

\_\_\_\_\_

c. (2 points) What is the smallest possible degree of the polynomial? Give a reason for your answer.

Degree: \_\_\_\_\_

Reason: \_\_\_\_\_

\_\_\_\_\_

d. (2 points) What are the zeros, and what is the smallest possible multiplicity of each?

\_\_\_\_\_

e. (2 points) Write a lowest-degree polynomial that has the graph in the previous problem. Note that  $f(0) = 6$ . (You may write in factored form)

\_\_\_\_\_

14. (4 points, 2 per part) You deposit \$5,000 at a bank that pays 7% interest. What is the balance in your account if the interest is compounded as follows: (SHOW THE CORRECT FORMULA AS PART OF YOUR ANSWER)

a.) Quarterly for 10 years (to the nearest \$0.01): \_\_\_\_\_

b.) Continuously for 10 years (to the nearest \$0.01): \_\_\_\_\_

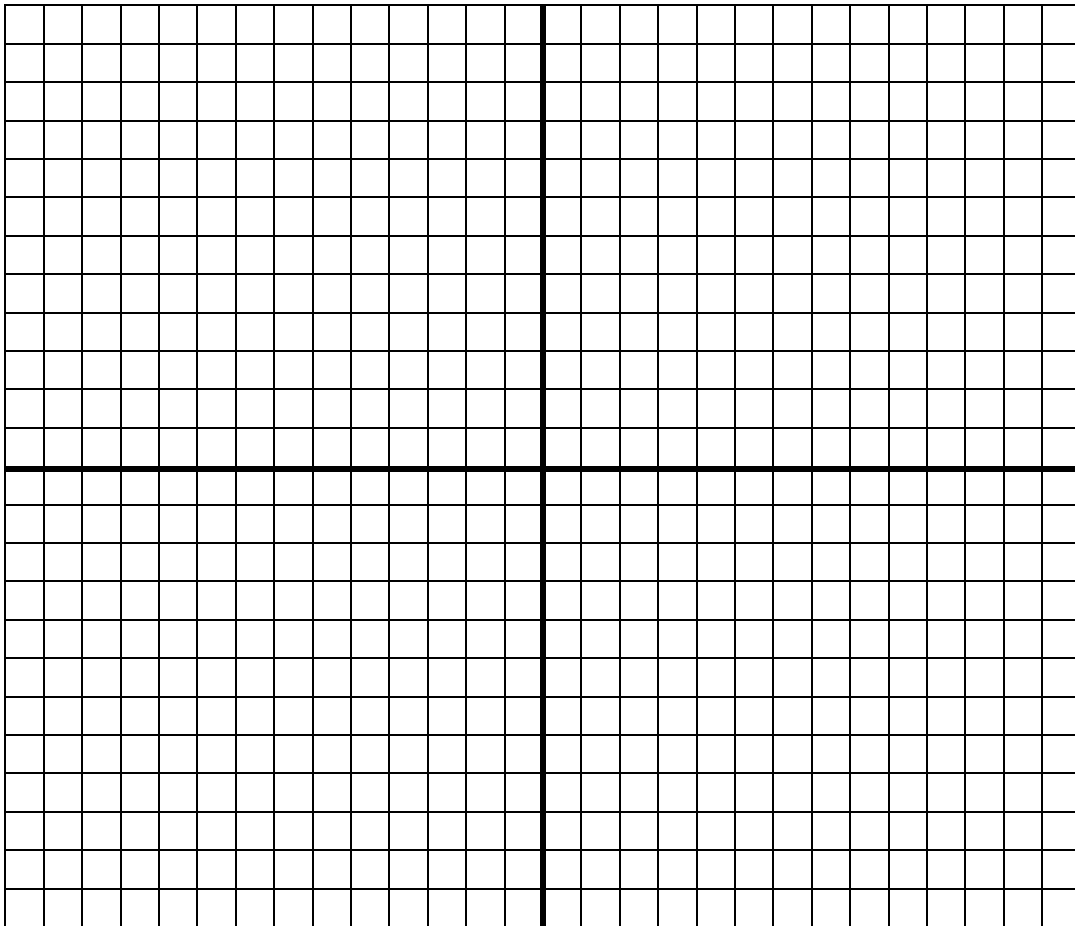
15. (5 points, as marked) For the function  $y = \frac{2x - 6}{x - 5}$

a. (2 points) Write the equations for any horizontal and vertical asymptotes.

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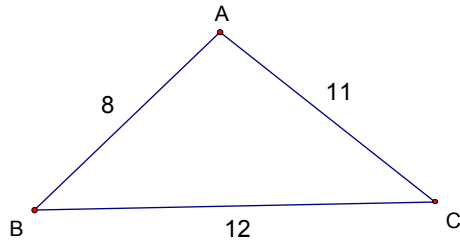
b. (3 points) Graph the function. The graph should include all vertical and horizontal asymptotes, zeros, holes, and y-intercepts (if they exist). Show the asymptotes as dotted lines. Label the zeros, holes, and y-intercept with (x,y) coordinates.

SCALE: 1 BLOCK = 1 UNIT



CALCULATORS IN DEGREE MODE—CHECK NOW!

16. (3 points, 1 per part) Solve triangle ABC. Express your answers to the nearest hundredth.



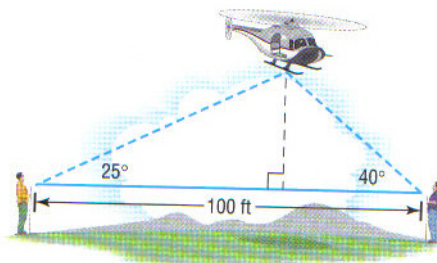
$\angle A =$  \_\_\_\_\_

$\angle B =$  \_\_\_\_\_

$\angle C =$  \_\_\_\_\_

17. (3 points) Two observers simultaneously measure the angle of elevation of a helicopter. One angle is measured as  $25^\circ$ , the other as  $40^\circ$ . (See figure below). If the observers are 100 feet apart and the helicopter lies over the line joining them, how high is the helicopter? Express your answer to the nearest hundredth.

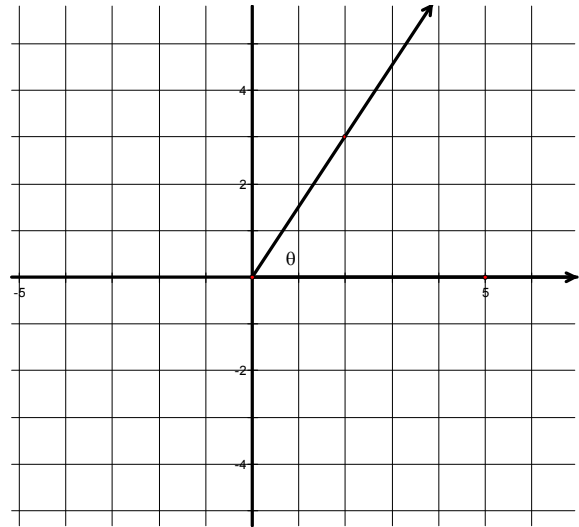
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CALCULATORS IN RADIAN MODE—CHECK NOW!

18. (3 points) The terminal side of angle  $\theta$  passes through the point (2, 3). What is the measure of angle  $\theta$  (in radians)? Express your answer to the nearest hundredth.

\_\_\_\_\_



19. (3 points) Prove the identity  $\frac{\tan(x)}{1 - \cos^2(x)} = \csc(x) \cdot \sec(x)$

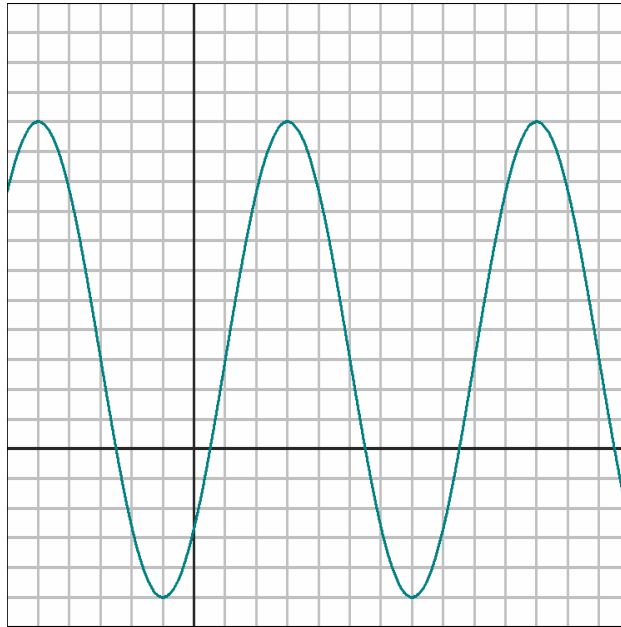
20. (3 points) Find all solutions of the equation  $\sin(2x) = \frac{\sqrt{2}}{2}$  (in radians) Express your answers to the nearest hundredth.

$x =$  \_\_\_\_\_

21. (4 points) Write the sinusoidal function represented by this graph.:

Horizontal Axis: SCALE 1 BLOCK = 1 RADIAN

Vertical Axis: SCALE: 1 BLOCK = 1 UNIT



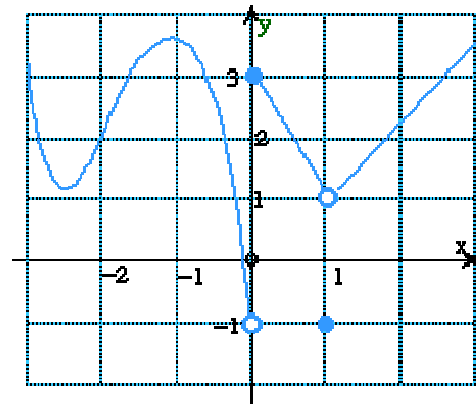
22. (4 points, 1 point each part) Use the graph of  $f(x)$  at the right to identify the following limits. If a limit is not a finite number, then give one of the following answers  $\infty$ ,  $-\infty$ . If no limit exists, write "no limit".

a)  $\lim_{x \rightarrow 0^+} f(x) =$  \_\_\_\_\_

b)  $\lim_{x \rightarrow 0} f(x) =$  \_\_\_\_\_

c)  $\lim_{x \rightarrow 1} f(x) =$  \_\_\_\_\_

d)  $\lim_{x \rightarrow +\infty} f(x) =$  \_\_\_\_\_

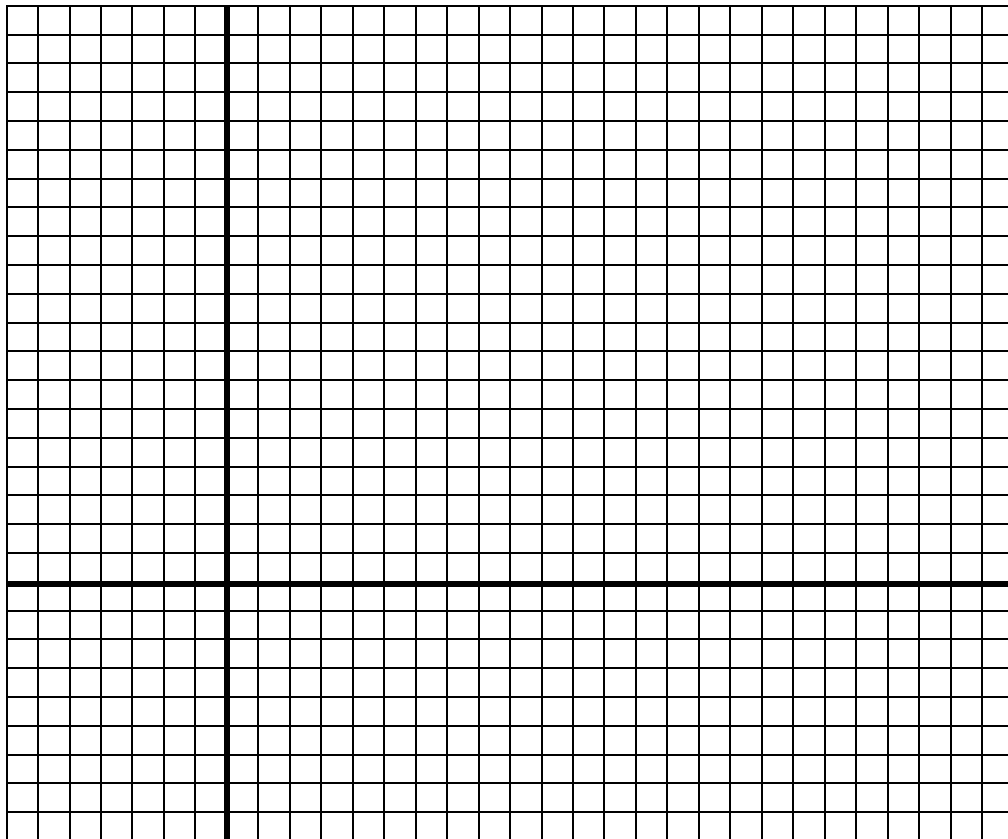


23. (6 points, 3 per part) In a tidal river, the depth of the water varies sinusoidally as a function of time. The time between high tide and low tide is 6 hours (therefore, a full cycle is 12 hours). At high tide the depth of the water at a certain dock is 18 feet, while at low tide the depth of the water is 6 feet. High tide occurs at 12:30 PM. Let  $d(t)$  represent the depth of the water in feet as a function of  $t$ , hours after 12 noon.

a) Write an equation for the function  $d(t)$ . Clearly show how you determined the 4 constant values in your equation.

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b) Sketch a graph of the depth of the water over time on the grid at right. Show at least one full cycle of the function. Clearly label the scale of your axes. CLEARLY LABEL (WITH  $(x, y)$  COORDINATES) THE “CRITICAL POINTS” IN THIS FULL CYCLE.



24. (3 points) How many ways can 2 goats, 3 cows, and 4 pigs be lined up so that animals of the same species are all next to each other? SHOW BOTH FORMULAS AND THE FINAL ANSWER.

Number of ways: \_\_\_\_\_

25. (4 points, 2 per part). Assume that enrollment at LHS is 500 each of freshmen, sophomores, juniors, and seniors (and assume the seniors are still here). (SHOW BOTH FORMULAS AND THE FINAL ANSWER).

a. If 4 students are selected at random, what is the probability they are 1 of each class? (Express your answer to the nearest hundredth.)

Probability: \_\_\_\_\_

b. If 4 are selected at random, what is the probability that none is a freshman? (Express your answer to the nearest hundredth.)

Probability: \_\_\_\_\_

26. (3 points) (DEGREE MODE) You are standing on the roof of a building 50 feet tall (see note below). There is a taller building 150 feet across the street. The angle of elevation from you to the top of the other building is  $42^\circ$ . The angle of depression from you to the base of the building is  $30^\circ$ . How tall is the other building (Express your answer to the nearest hundredth.)? (INCLUDE A DIAGRAM AS PART OF YOUR SOLUTION)

Height of building: \_\_\_\_\_

NOTE: the words “50 feet tall” in the first sentence were included by mistake. They should be disregarded by anyone using this test for review or practice. Different answers were accepted as correct on the original exam in June, 2005.