Name **Answer Key**

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Class block (circle): C D E G H

Lexington High School
Mathematics Department
Honors Algebra 2
Final Exam 2005

This is a 90-minute exam, but you will be allowed to work for up to 120 minutes.
Graphing calculators are permitted.

The exam has 3 parts. Directions for each part appear below. In total, there are 99 points that you can earn. A letter grade scale will be set by the course faculty after the tests have been graded.

Part A. Short Answer Questions

8 questions, 4 points each, 32 points total

You must write your answers in the answer boxes.

If your answer is correct, you will receive full credit. Showing work is not required.

If your answer is incorrect, you may receive half credit if you have shown enough correct work.

Part B. Written Response Problems

4 multi-part problems, 10 points each, 40 points total

Write a complete, clearly explained solution to each problem. If you use your calculator for a significant step, explain what you did on the calculator. Partial credit will be given.

Part C. Multiple Choice Questions

9 questions, 3 points each, 27 points total

Circle the letter in front of the correct answer to each question.

Part A. Short Answer Questions

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1. Evaluate $\log_5(\log_3(\log_4 32 + \log_4 2))$.

$\log_4 32 + \log_4 2 = \log_4 64 = 3$
$\log_3(3) = 1$
$\log_5(1) = 0$

Answer to question 1:

0

2. Perform this matrix multiplication: $\begin{bmatrix} 1 & -5 & 3 \\ m & 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ n \end{bmatrix}$

$$\begin{bmatrix} 1 & -5 & 3 \\ m & 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ n \end{bmatrix} = \begin{bmatrix} 1\cdot 2 + (-5)\cdot 4 + 3\cdot n \\ m\cdot 2 + 0\cdot 4 + 6\cdot n \end{bmatrix} = \begin{bmatrix} -18 + 3n \\ 2m + 6n \end{bmatrix}$$

Answer to question 2:

$$\begin{bmatrix} -18 + 3n \\ 2m + 6n \end{bmatrix}$$

3. Using any method, find a quadratic function $F(x)$ such that $F(0) = 6$, $F(3) = 3.84$, and $F(5) = 4.8$.

(0,6) $c = 6$

(3, 3.84) $9a + 3b + 6 = 3.84 \rightarrow 3a + b = -.72$

(5, 4.8) $25a + 5b + 6 = 4.8 \rightarrow 5a + b = -.24$

$2a = .48 \rightarrow a = .24 \quad b = -.72 - 3(.24) = -1.44$

Answer to question 3:

$F(x) = .24x^2 - 1.44x + 6$

4. A monkey types a random sequence of 6 capital letters (from the English alphabet of A to Z). What is the probability that the 6 letters are all different?

$$\frac{{}_{26}P_6}{26^6} = \frac{26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 21}{26^6} = .5366$$

Answer to question 4: (An answer rounded to the nearest 0.001 is acceptable.)

P(6 different) = 0.537 or 53.7%

5. Write the series $\sum_{k=1}^{5}(k^2 - k)$ using addition, then find the sum of the series.

Answer to question 5:

$\sum_{k=1}^{5}(k^2 - k)$ = $(1^2\text{-}1) + (2^2\text{-}2) + (3^2\text{-}3) + (4^2\text{-}4) + (5^2\text{-}5)$ = 0 + 2 + 6 + 12 + 20 = __40__.

6. Find values of A and B such that this linear system would have infinitely many solutions.

$5x + Ay = 24 \Rightarrow 10x + 2Ay = 48$
$2x - 4y = B \Rightarrow 10x - 20y = 5B$

$-20 = 2A$ $\quad$ $5B = 48$

$-10 = A$ $\quad$ $B = 9.6$

Answer to question 6:

A = __–10___ B = __9.6_____

7. Here is a data set of two-digit integers presented as a stem-and-leaf plot.

7	1
6	2 5 5
5	4
4	2 3 9
3	2 8
2	4 6
1	2 9

Find the Interquartile Range (IQR) of this data set without using a calculator, and briefly explain how you found it.

Answer to question 7:

The IQR is __36___, and here is how I found it:

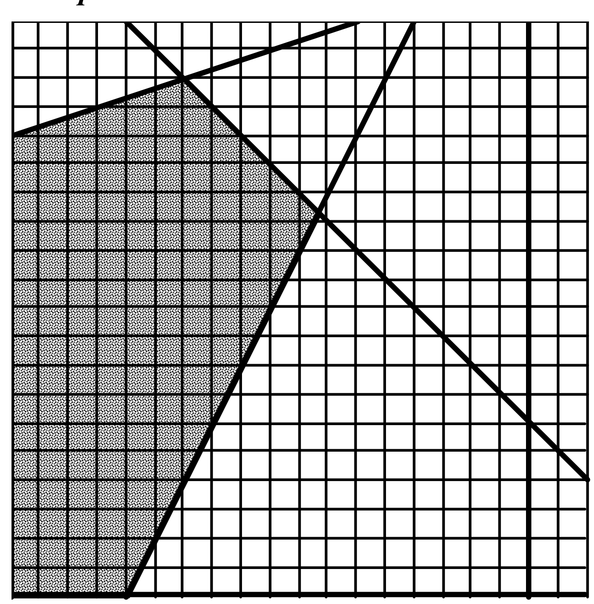
There are 14 data points, 7 in the first half and 7 in the second half. The fourth point from the beginning is Q1. The fourth point from the end is Q3. The IQR is Q3 - Q1 or 62 - 26 which equals 36.

8. A two-variable linear programming problem has the following constraints:

$x \geq 0$
$y \geq 0$
$x \leq 18$
$x + y \leq 24$
$2x \leq y + 8$
$48 + x \geq 3y$

On the grid, draw and shade the feasible region. **Some** of the lines you will need have already been drawn for you.

Answer to question 8:



The window for this grid is $0 \leq x \leq 20$, $0 \leq y \leq 20$.

Part B. Written Response Problems

4 multi-part problems, 10 points each, 40 points total

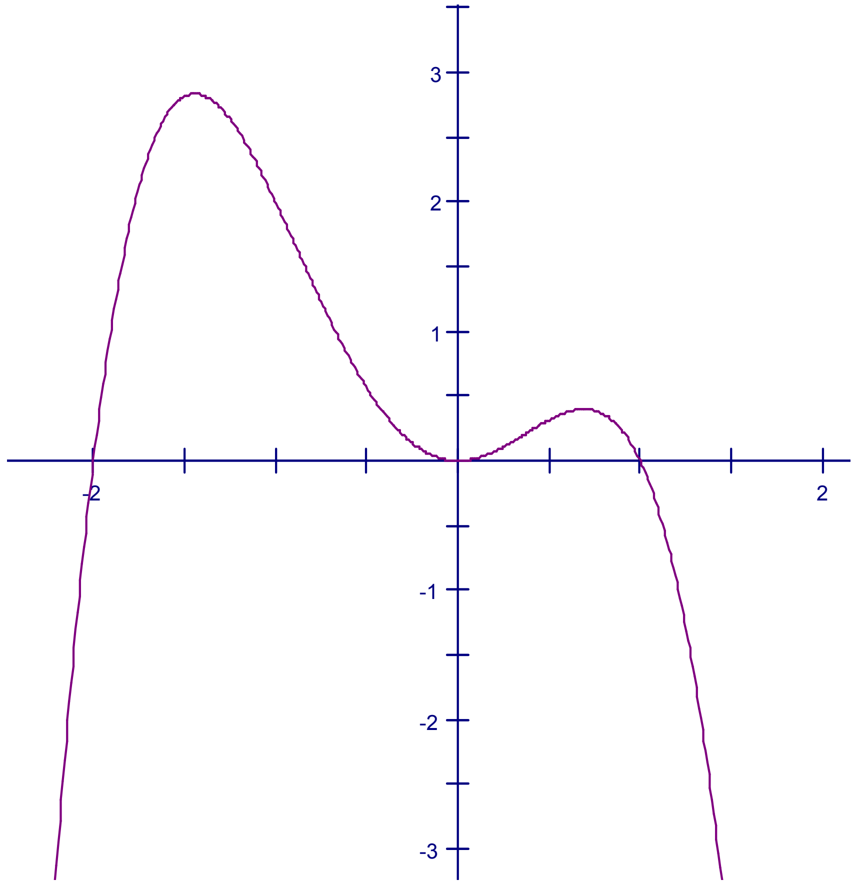
Write a complete, clearly explained solution to each problem. If you use your calculator for a significant step, explain what you did on the calculator. Partial credit will be given.

9. A graph of polynomial function $P(x)$ is shown.

The domain of $P(x)$ is the set of real numbers.

The graph is tangent to the x-axis at (0, 0).

All zeros of $P(x)$ can be observed on this graph.



a. Could $P(x)$ be a third-degree polynomial? Explain why or why not.

No. Two possible explanations:

- **Although $P(x)$ has 3 zeros, one of them has a multiplicity of 2 so the degree must be at least 4.**
- **The end behavior of the graph (downward on both the left and the right) is that of an even-degree function, so $P(x)$ cannot have degree 3.**

b. Is the leading coefficient of $P(x)$ positive or negative?

negative

c. Write a possible equation for $P(x)$.

$P(x) = -(x + 2)(x^2)(x - 1) = -x^2(x^2 + x - 2)$

d. What is the range of $P(x)$?

$y \le 2.8$ ***(OK to estimate the y-coordinate of the maximum point)***

10. When rabbits were first brought to Australia, they had no natural enemies so their numbers increased rapidly. Suppose that the population grew from 60,000 rabbits in year 1865
to 480,000 rabbits in year 1867. Assume that the growth of this rabbit population can be modeled with an exponential function.

a. Let x = the number of years elapsed since 1865. Without using your calculator, find the exponential function $P(x)$ that models the rabbit population. Show your work.

1865: 60,000 (0, 60000)

1867: 480,000 (2, 480000)

$r^{2-0} = 480000 / 60000 = 8$

$r = 2\sqrt{2}$

$$P(x) = 60{,}000 \cdot \left(2\sqrt{2}\right)^x$$

b. Predict the number of rabbits for the year 1868.

$$P(3) = 60{,}000 \cdot \left(2\sqrt{2}\right)^3 \approx 1{,}357{,}645$$

c. In what year would the rabbit population first exceed 10,000,000?

$$P(n) = 60{,}000 \cdot \left(2\sqrt{2}\right)^n > 10{,}000{,}000$$

$$\left(2\sqrt{2}\right)^n > 166.\overline{6}$$

$$n > \frac{\log(166.\overline{6})}{\log(2\sqrt{2})} = 4.92 \approx 5$$

so the year is 1870

d. Suppose that all of these rabbits were descendents of the first pair of rabbits brought to Australia. Based on the model, find the date that these first rabbits arrived in Australia.

$$2 = 60{,}000 \cdot \left(2\sqrt{2}\right)^x \qquad x = \frac{\log\left(\frac{2}{60{,}000}\right)}{\log(2\sqrt{2})} = -9.9$$

so the year is 1855

11. For all questions on this page, let $f(x) = \dfrac{5}{x-4}$, $g(x) = x^2$, and $h(x) = (3x)^2$.

a. Find $f^{-1}(30)$.

$$x = \frac{5}{f^{-1}(x)-4} \qquad f^{-1}(x) = \frac{5}{x}+4 \qquad f^{-1}(30) = \frac{5}{30}+4 = 4\tfrac{1}{6}$$

b. Identify the domain of the composite function $f(g(x))$.

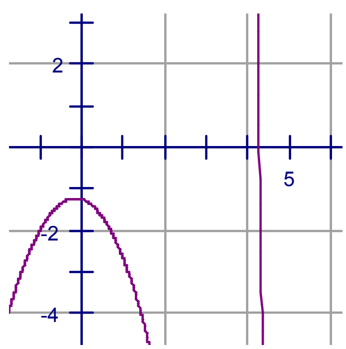
$$f(g(x)) = \frac{5}{x^2-4} = \frac{5}{(x+2)(x-2)}$$

so the domain is all real numbers except 2 and –2

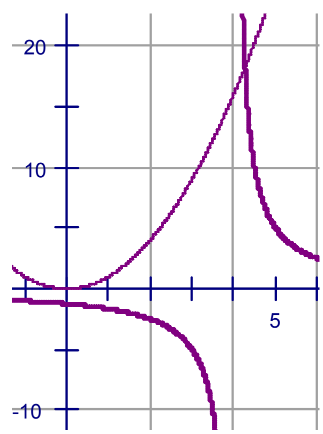
c. Find the zero(s) of the difference function $f - g$.

$$f(x) - g(x) = 0$$
$$\frac{5}{x-4} - x^2 = 0$$

$$f(x) = g(x)$$
$$\frac{5}{x-4} = x^2$$



x = 4.274



d. Describe the transformation needed to transform the graph of $g(x)$ into the graph of $h(x)$.

Horizontal compression by 3 ~ or ~ Vertical stretch by 9

12. A wooden cube has three red faces, two green faces, and one blue face.

a. Consider a single roll of this cube. Define these events:

E = rolling red
F = not rolling blue

Are events E and F independent? Justify your answer.

No. Three ways to justify:

- $P(E) = \frac{3}{6}$, but $P(E \mid F) = \frac{3}{5}$. Since $P(E) \neq P(E \mid F)$, the outcome of event F affects the likelihood of event E, so the events are not independent.
- $P(F) = \frac{5}{6}$, but $P(F \mid E) = 1$. Since $P(F) \neq P(F \mid E)$, the outcome of event E affects the likelihood of event F, so the events are not independent.
- $P(E \text{ and } F) = \frac{3}{6}$, while $P(E) \cdot P(F) = \frac{3}{6} \cdot \frac{5}{6}$. Since $P(E \text{ and } F) \neq P(E) \cdot P(F)$, the events are not independent.

b. Suppose that you rolled the cube twice. Find the probability that you would get the same color both times.

P(RR) + P(GG) + P(BB) = $\frac{3}{6} \cdot \frac{3}{6} + \frac{2}{6} \cdot \frac{2}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{14}{36} \approx 0.389.$

c. Suppose that you rolled the cube 10 times. Find the probability that you would roll blue exactly 3 out of the 10 times.

$${}_{10}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7 \approx 0.155$$

Part C. Multiple Choice Questions

9 questions, 3 points each, 27 points total

Circle the letter in front of the correct answer to each question.

13. Which of the following is not a pair of inverse functions? **(C)**

(A) $f(x) = x + 7$ and $g(x) = x - 7$

(B) $f(x) = 4x$ and $g(x) = \frac{1}{4}x$

(C) $f(x) = x^3$ and $g(x) = x^{-3}$

(D) $f(x) = \sqrt[3]{x-1}$ and $g(x) = x^3 + 1$

(E) $f(x) = e^x$ and $g(x) = \ln x$

14. Which of the following is equal to $\frac{\sqrt[3]{x}}{\sqrt[4]{x}}$? **(D)**

(A) $\sqrt[\frac{3}{4}]{x}$

(B) $\sqrt[-1]{x}$

(C) $x^{4/3}$

(D) $x^{1/12}$

(E) x^{-12}

15. Which of the following equations has a graph with a vertex at $(n, 4)$? **(C)**

(A) $y - n = 2(x - 4)^2$

(B) $(x - n)^2 + (y - 4)^2 = 1$

(C) $y = x^2 - 2nx + n^2 + 4$

(D) $y - 4 = 3(x + n)^2$

(E) $y = 3(x - n)^2 - 4$

16. Augmented matrices may be used to solve systems of linear equations. In doing so, which of the following would **not** be a valid operation to perform on an augmented matrix? **(B)**

(A) Interchange two rows.

(B) Add the same non-zero number to every entry in a row.

(C) Multiply every entry in a row by the same positive number.

(D) Multiply every entry in a row by the same negative number.

(E) Subtract a multiple of one row from another row.

17. The weights of adult Newfoundland dogs have approximately a normal distribution, with a mean of 140 pounds and a standard deviation of 20 pounds. Suppose that a Newfoundland is considered overweight if it weighs more than 170 pounds. What percentage of these dogs are overweight? **(B)**

(A) about 5%

(B) about 7%

(C) about 10%

(D) about 20%

(E) about 30%

18. Express $\left(\frac{2x^{-3}y^{5}}{x^{5}y^{-1}}\right)^{-2}$ in simplest form: **(E)**

(A) $\left(\frac{1}{4x^{8}y^{4}}\right)$

(B) $\left(\frac{x^{16}y^{12}}{4}\right)$

(C) $\left(\frac{2x^{2}}{y^{4}}\right)$

(D) $\left(\frac{-4}{x^{9}y^{6}}\right)$

(E) $\left(\frac{x^{16}}{4y^{12}}\right)$

19. Find the center of the circle whose equation is $x^2 + y^2 + 8x - 12y + 43 = 0$. **(B)**

(A) (4, –6)

(B) (–4, 6)

(C) (8, –12)

(D) (–8, –12)

(E) none of the above

20. A sequence is defined recursively as follows: $t_1 = 5$; $t_n = \frac{1}{2}\,t_{n-1}$ for $n > 1$.
Which of the following is an explicit formula for this sequence? **(D)**

(A) $t_n = 5 + \frac{1}{2}(n - 1)$

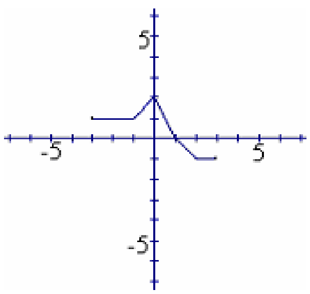
(B) $t_n = 5 \cdot \frac{1}{2}(n - 1)$

(C) $t_n = 5 + (\frac{1}{2})^{(n-1)}$

(D) $t_n = 5 \cdot (\frac{1}{2})^{(n-1)}$

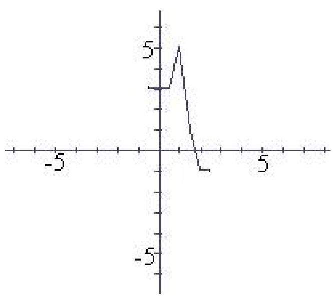
(E) none of the above

21. Here is the graph of a function $f(x)$.

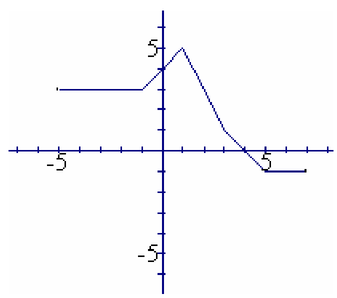


Which of the following is the graph of $g(x) = 2\,f(0.5(x - 1)) + 1$? **(B)**

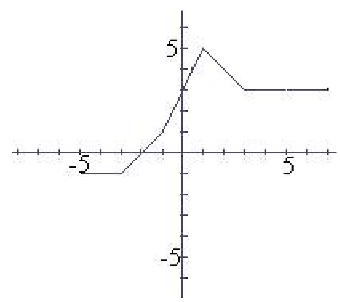
(A)



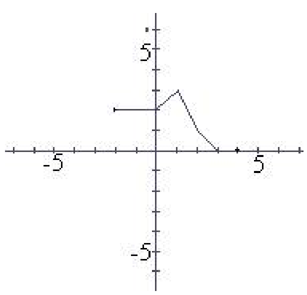
(B)



(C)



(D)



(E)