

Algebra 2
Level 2

Final Exam
June 2003

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Name _____

Directions:

Formulas are provided in each section of the exam and there is also a formula sheet on the last page which you may tear off and use.

Be sure to show work so that we can give you partial credit.

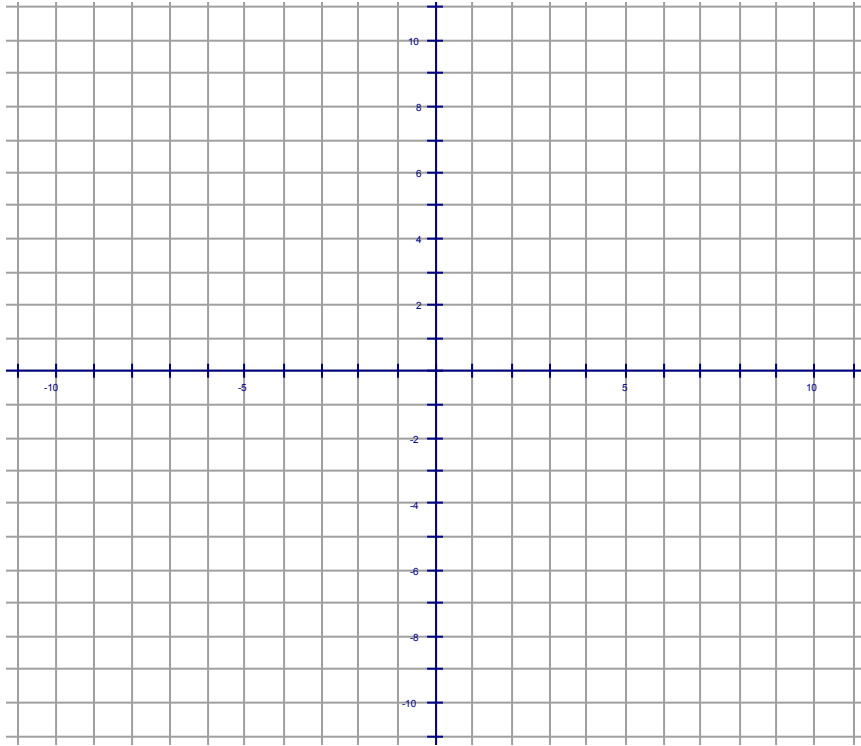
Good Luck! Have a great summer.

Linear Algebra

Slope formula. If (x_1, y_1) , (x_2, y_2) are two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Equation for a line. The equation for any line is of the form $y = mx + b$
where m = the slope and b = the y intercept.

1. Sketch a graph of the line $y = -\frac{1}{2}x + 2$. Show at least 2 points.



2. A line contains the points $(3, 14)$ and $(8, 16)$.
- a. What is the slope of this line?

slope _____

- b. Write an equation for the line.

equation for line _____

3. Solve these inequalities and graph your answers on a number line.

a. $3x + 4 < 28$

b. $-2x > 16$

c. $25 \leq 2x + 3 \leq 39$

4. Find the point of intersection of these lines.

$y = 7x + 3$

$x =$

$y = 5x - 2$

$y =$

5. Alfonso earns \$14 an hour and makes car payments of \$350 a month.

Write an equation showing how much money (y) he has left at the end of the month depending on how many hours (x) he works.

equation _____

6. An oven starts at room temperature, $68\frac{1}{4}$, and heats up $18\frac{1}{4}$ per minute when it is turned on.

a. If y = temperature and x = minutes since the oven is turned on, write a linear $y = mx + b$ equation to relate these two variables.

equation _____

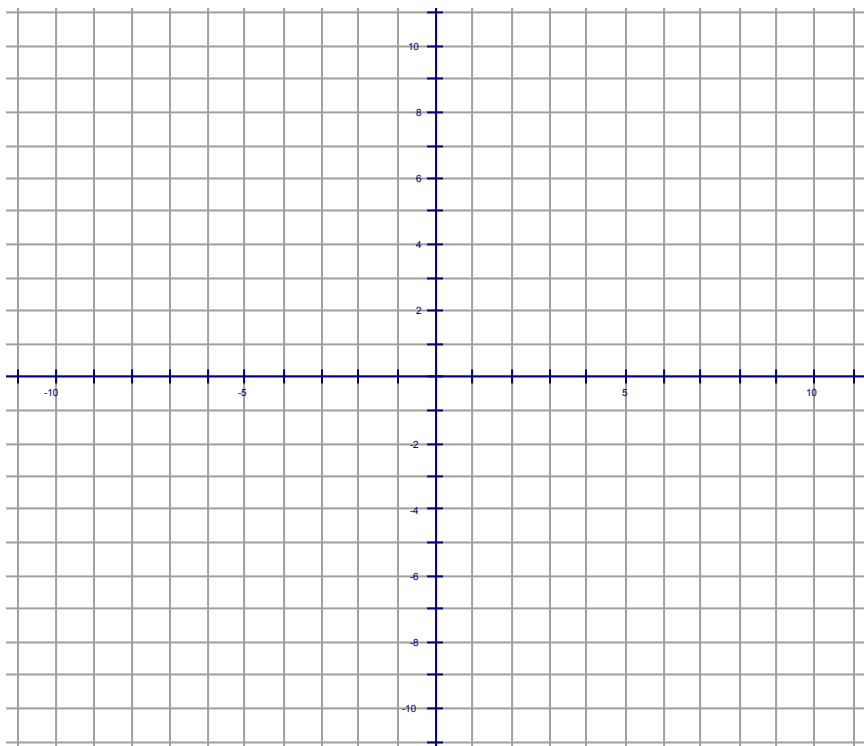
b. If the oven is heated for 5 minutes how hot will it be?

temperature after 5 min. _____

c. How long will it take to reach a temperature of $350\frac{1}{4}$?

time to reach $350\frac{1}{4}$ _____

7. Graph $y \geq 2x - 4$



8. You can buy peanuts for \$4 a pound and raisins for \$2 a pound. Suppose you have \$40 and can spend some or all of it on peanuts and raisins. Write an inequality expressing this fact. Let x = pounds of peanuts and y = pounds of raisins.

inequality _____

Quadratics

The Equation for falling objects. $y = -16x^2 + V_0x + H_0$.

x = time in seconds and y = height in feet.

V_0 is the starting speed of the object in feet per second.

H_0 is the starting height in feet.

Quadratic formula: Given the equation $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

1. ANSWER ANY TWO. Solve by any method you like: graphing, quadratic formula, factoring or completing the square.

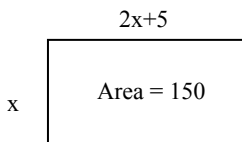
a. $5x^2 - 6x - 100 = 0$ $x =$

b. $4x^2 - 144 = 0$ $x =$

c. $x^2 + 57x = 0$ $x =$

d. $3x^2 = 57x$ $x =$

2. A rectangle has an area of 150, a width of x and a length of $2x+5$. Find x .



$x =$

3. A rocket is fired from the ground at 400 feet per second.
(Starting height = 0 and starting speed = 400.)

- a. Use the equation for falling objects and write an equation.
(If you are going to solve this by graphing you will need a good graph.)

equation _____

- b. When does the rocket hit the ground?

time rocket hits the ground _____

- c. How high does the rocket go and when does it get this high?

maximum height _____

time the height is reached _____

Matrices

1. Perform this matrix arithmetic using your calculator or by hand. If any are impossible say so.

a.

$$5 \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 6 & 1 \\ -5 & 0 \end{bmatrix} =$$

b.

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 5 & 8 \end{bmatrix} =$$

c.

$$\begin{bmatrix} 1.5 & 2.3 & 5.8 \\ 0.6 & 1.9 & 2.2 \\ 1.4 & 3.7 & 4.2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ 1 & 9 \\ 8 & 3 \end{bmatrix} =$$

2. Solve this matrix equation.

$$\begin{bmatrix} 3 & x \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ y \end{bmatrix}$$

$$x = \quad y =$$

3. Solve this system of equations by writing a matrix equation and using your calculator or any other way.

$$\begin{aligned} 6x + 2y &= 6 \\ -4x + 5y &= -23 \end{aligned}$$

$$x = \quad y =$$

4. A company sells vinegar and olive oil at two different stores, one in Boston and one in Lynn. At the Boston store they have 75 bottles of vinegar and 66 of oil and at Lynn they have 48 bottles of vinegar and 19 of oil. The matrix below represents how much of each is on hand at each store.

$$\begin{array}{l} \text{Vinegar} \\ \text{Oil} \end{array} \begin{array}{c} \text{Bos. Lynn} \\ \begin{bmatrix} 75 & 48 \\ 66 & 19 \end{bmatrix} \end{array}$$

The company sells vinegar for \$4.95 a bottle and oil for \$8.50 the matrix below represents the prices of these products.

$$\begin{array}{l} \text{Vinegar Oil} \\ \$ \end{array} \begin{array}{c} \\ \begin{bmatrix} 4.95 & 8.50 \end{bmatrix} \end{array}$$

a. Write the two matrices in the proper order so that they can be multiplied and perform the multiplication. Only one way works.

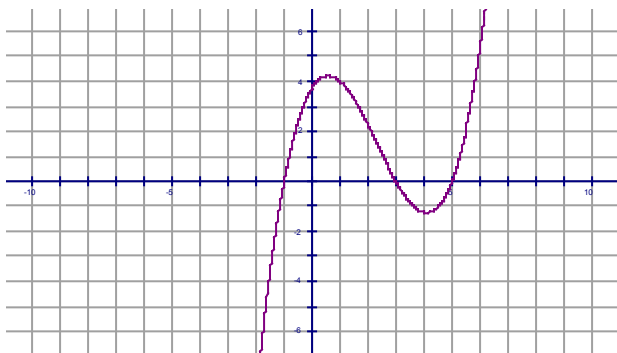
b. What does the resulting matrix represent in the real world?

Polynomials

1. Multiply out and collect like terms:

$$y = 5x(x+1)(x-3)$$

2. Choose the polynomial equation that fits this graph.



a. $y = 5x + 4$

c. $y = 0.25(x+1)(x-3)(x-5)$

b. $y = -3(x+1)(x-3)(x-5)$

d. $y = (x-1)(x+3)(x+5)$

Exponential Equations

Basic exponential equation: $y = Ab^x$ A = starting value b = base x = exponent

Interest formula.

$$y = P(1 + I)^x$$

P = starting amount of money, known as your Principal..

I = interest rate. It is negative if you are paying interest or losing money.

If I = 7% you use 0.07 here.

x = number of years. y = final amount of money.

Population growth or decline.

$$y = P(1 + I)^x$$
 P = starting population.

I = growth rate. If it is negative the population is declining.

x = number of years. y = final population.

1. The population of India is now 1.1 billion people. If the population grows every year at 1.3% per year:

a. What will it be in 20 years?

population in 20 years _____

b. When will it double to 2.2 billion? Solve by graphing.

number of years until population becomes 2.2 billion _____

2. If you invest \$5000 in a bank that pays 4% yearly interest:

a. How much will you have after 15 years?

amount after 15 years _____

b. When will you have \$15,000?

years until \$15,000 _____

3. A car loses 20% in value every year. If the starting value is \$6,000 how much will it be worth in 5 years? (Use a negative growth rate.)

value after 5 years _____

4. Solve.

a. $x^7 = 914$

x =

b. $2(x)^4 = 10$

x =

c. $6^x = 425$

x =

Rational functions

1. Match each equation with its graph.

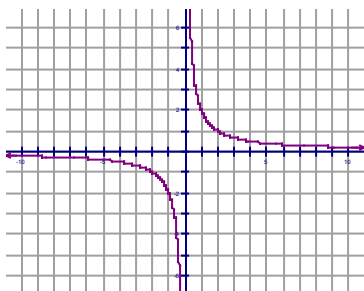
$$y = \frac{2}{x}$$

$$y = \frac{-2}{x-3}$$

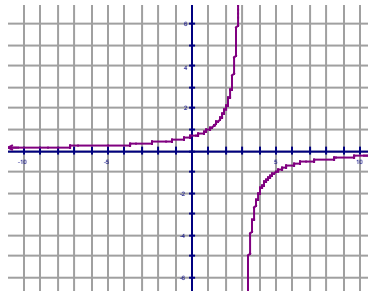
$$y = \frac{2}{x-3}$$

$$y = \frac{2}{x} + 3$$

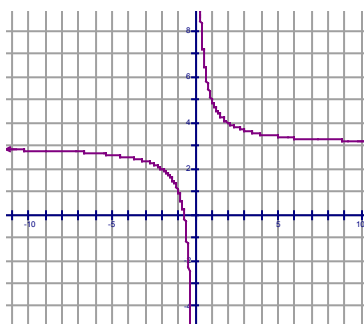
a.



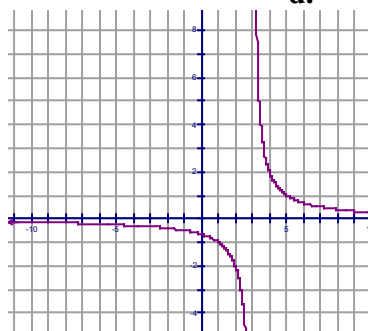
b.



c.



d.



Probability and Statistics

Multiplication principle.

If one of K objects must be chosen and one of M other objects must be chosen and one of N other objects must be chosen then there are KMN ways to do this.

Permutations (order counts).

If you arrange n objects taken r at a time there are: ${}^n P_r = \frac{n!}{(n-r)!}$ ways to do this.

Combinations (order does not count).

If you want to choose any r out of n objects there are ${}^n C_r = \frac{n!}{(n-r)!r!}$ ways to do this.

The fundamental rule of probability.

If there are n equally likely ways that an event can occur then the probability of any one of k of them happening is $P = \frac{n}{k} = \frac{\text{number of ways wanted}}{\text{total number of ways}}$.

Independent events.

If events A and B are independent of each other, so that one happening has no effect on the other then

$$\text{Prob}(A \text{ and } B \text{ both happen}) = \text{Prob}(A \text{ happens}) \times \text{Prob}(B \text{ happens})$$

Measures of the Center of Data

Mode The most common value in a set.

Median The middle or center value when data is arranged in numerical order.

If there is an even number of pieces of data average the two in the middle.

Mean The measure commonly called the average, found by adding all the values in a set and dividing by the total number of values.

-
1. In how many ways can the first three positions occur in a horse race with 11 horses?
 2. Suppose a hand of 5 cards are dealt from a well shuffled deck of 52 cards. How many possible hands are there?

number possible _____

3. How many 5 character license plates are possible which go letter-letter-numeral-numeral-numeral, where a numeral is any number from 0 to 9, and any letter or numeral can be repeated?

number possible _____

4. Two dice are rolled. What is the probability that the numbers shown add up to 3?

probability _____

5. There are two bags of marbles. The first bag contains 8 marbles: 3 red and 5 blue. The second bag contains 20 marbles: 11 red and 9 blue. If a marble is drawn from each bag what is the probability that both are red?

probability _____

6. The heights of children at a birthday party are taken. Find the mean, mode and median height.

The heights in inches are {37, 39, 39, 40, 42, 44, 53}

mean height =

mode height =

median height =

7. The number of hot dogs consumed at a hot dog eating contest by each of 50 contestants is tabulated below. What is the mean (average) number of hot dogs eaten and the median number eaten?

Number of hot dogs eaten	Number of contestants
--------------------------	-----------------------

10	3
----	---

11	11
----	----

12	19
----	----

13	16
----	----

14	0
----	---

15	0
----	---

16	<u>1</u>
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Total	50
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average number eaten _____

median number eaten _____

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