

Name \_\_\_\_\_

4/27/05

Answer 3 of the following 4 open response problems. Problems will be graded in the same manner as AP open response problems. In the spaces below, indicated which 3 problems you would like graded. Only the problems you indicate below will be graded.

I would like problems \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ graded.

**Problem 1**

Consider the curve given by  $xy^2 - x^3y = 6$

a. Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

b. Find all points on the curve whose  $x$ -coordinate is 1 and write an equation for the tangent line at each of these points.

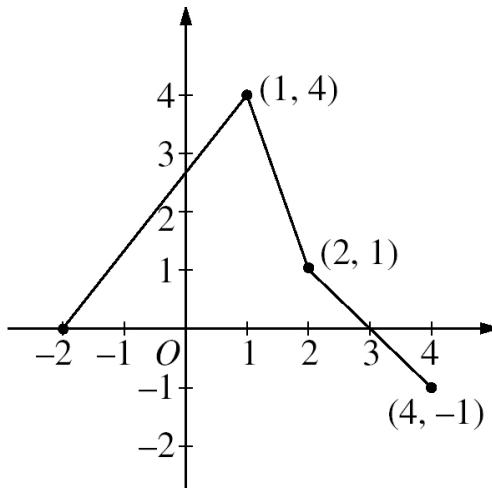
c. Find the  $x$ -coordinate of each point on the curve where the tangent line is vertical.

**Problem 2**

The graph of the function,  $f$ , consisting of three line segments, is given below.

Let  $g(x) = \int_1^x f(t) dt$ .

a. Compute  $g(4)$  and  $g(-2)$ .



b. Find the instantaneous rate of change of  $g$ , with respect to  $x$ , at  $x = 1$ .

c. Find the absolute minimum value of  $g$  on the closed interval  $[-2, 4]$ . Justify your answer.

d. The second derivative of  $g$  is not defined at  $x = 1$  and  $x = 2$ . Which of these values are  $x$ -coordinates of points of inflection of the graph of  $g$ ? Justify your answer.

## Problem 3

$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function of time  $t$ . The table above shows the rate as measured every 3 hours for a 24 hour period.

- a. Use a midpoint Riemann sum with 4 equal sub-divisions to approximate  $\int_0^{24} R(t) dt$ .  
Using correct units, explain the meaning of your answer in terms of water flow.

- b. Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.

- c. The rate of water flow  $R(t)$  can be approximated by the quadratic function  $Q(t) = \frac{1}{79}(768 + 23t - t^2)$ . Use  $Q(t)$  to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

**Problem 4**

Consider region **R** bounded by the line  $y = x - 1$ , the parabola  $y^2 = 2x + 6$ , and the  $x$ -axis.

a. Draw a detailed sketch of region **R**.

b. Find the area of region **R**.

c. Find the volume of revolution if region **R** is rotated around the line  $x = 5$ .