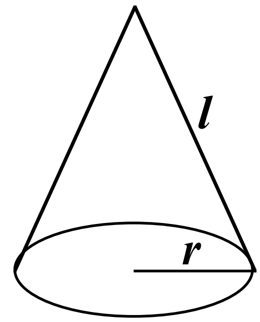
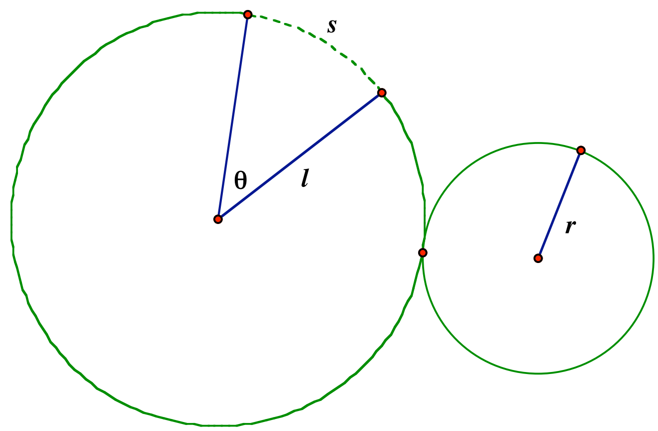
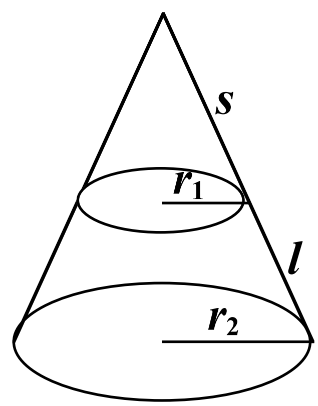



The lateral area formula for a cone comes from the fact that the lateral area of a cone is made of a circle where a wedge has been cut out to make it fit around the base. It also helps to remember the definition of a radian – one radian is the angle needed to make a piece of arc be the same length of the radius of the circle, thus $\theta = \frac{s}{l}$ ( where $s$ is the arc length and $l$ is the radius of the circle, see the large circle below).

$$
\begin{aligned}
\text{Cone Lateral Area } &= \pi \cdot l^2 - \frac{\theta}{2\pi}\left(\pi \cdot l^2\right) \\
&= \pi \cdot l^2\left(1 - \frac{\theta}{2\pi}\right) \\
&= \pi \cdot l^2\left(1 - \frac{s}{2\pi \cdot l}\right) \\
&= \pi \cdot l^2\left(1 - \frac{2\pi \cdot l - 2\pi \cdot r}{2\pi \cdot l}\right) \\
&= \pi \cdot l^2\left(1 - \left(1 - \frac{r}{l}\right)\right) \\
&= \pi \cdot l^2\left(\frac{r}{l}\right) \\
&= \pi \cdot l \cdot r
\end{aligned}
$$




The lateral area formula for a frustum is based on the lateral area of the cone, with the top removed. It takes advantage of the fact that the cone removed is similar to the whole cone.




$$
\begin{aligned}
\text{Frustum Lateral Area } &= \pi \cdot r_2(s + l) - \pi \cdot r_1 s \\
&= \pi(r_2 s + r_2 l - r_1 s) \\
&= \pi\left(r_2 l + (r_2 - r_1)s\right) \\
&= \pi\left(r_2 l + r_1 l\right) \quad \text{(see below for this substitution)} \\
&= \pi\left(r_2 + r_1\right)l
\end{aligned}
$$

$$
\begin{aligned}
\frac{r_1}{s} &= \frac{r_2}{s + l} \\
r_1(s + l) &= r_2(s) \\
r_1 \cdot l &= r_2 \cdot s - r_1 \cdot s \\
r_1 \cdot l &= s(r_2 - r_1)
\end{aligned}
$$

With our calculus calculation, as $r_2 \to r_1 \to r$ then $= \pi\left(r_2 + r_1\right)l$ goes to $= 2\pi \cdot rl$.