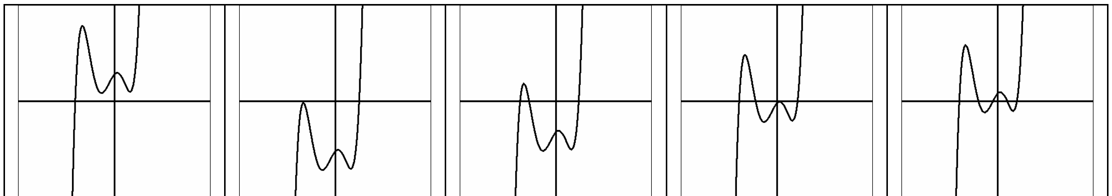
Part A. Zeroes of polynomials

1. 0 zeroes is impossible. For any degree 5 polynomial, being an odd-degree polynomial, the graph must have end behavior that is "up" on one side and "down" on the other side. Since the graph is continuous, it must cross the x-axis, so it must have a zero.

6 zeroes is impossible. The number of zeroes of a polynomial is never greater than the degree.



1 zero	2 zeroes	3 zeroes	4 zeroes	5 zeroes

2. **a.** ±1, ±2, ±1/3, ±2/3

b. –2/3 is a zero, because $f(-2/3) = 3 \cdot (-2/3)^3 - (-2/3)^2 - 5(-2/3) - 2 = 0$.

c. There are two irrational zeroes, because the graph crosses the x-axis at $x \approx -0.618$ and at $x \approx 1.618$, and neither of these values is on the list of possible rational zeroes.

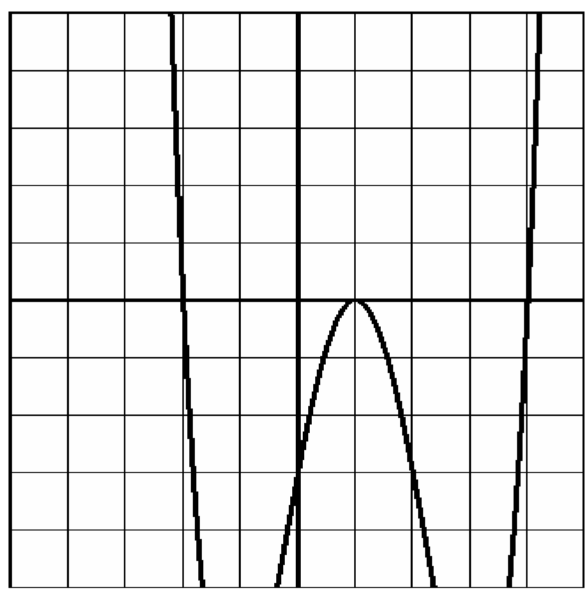
Part B. Graphs of polynomials

1. Two important implications of $\dfrac{P(x)}{x+2}$ being a cubic polynomial are:

- $P(x)$ must be a 4th degree polynomial.
- $(x + 2)$ is a factor of $P(x)$, and therefore $x = -2$ is a zero.

So $P(x)$ has zeroes of –2, 1, and 4. The zero at 1 has multiplicity 2 so the graph must be tangent to the x-axis at $x = 1$.

Here's a graph that satisfies all the requirements. (It's a "curvy W" shaped graph but the minimum points are below the bottom of the grid. OK if you drew your "W" with the minimum points appearing on the grid.)



2. $Q(x) = -\frac{1}{9}\, x\,(x+2)^3$

Part C. Division and remainders

1. You must use long division (synthetic division doesn't work when the divisor is not linear).
You should get a quotient of $(3x + 1)$ and a remainder of 0.
Since the remainder is zero, $x^2 - 3x + 3$ **is a factor** of $3x^3 - 8x^2 + 6x + 3$.

2. The division from problem **1** gives that $3x^3 - 8x^2 + 6x + 3 = (x^2 - 3x + 3)(3x + 1)$.

Applying the Quadratic Formula to $(x^2 - 3x + 3)$ shows that this quadratic has no real zeroes, because the result will involve the square root of a negative number.

The factor $(3x + 1)$ has $x = -1/3$ as a zero.

Answer: The only real zero is $x = -1/3$. [If you used the Quadratic Formula to find two other zeroes in the complex number system, that's fine, but it wasn't required by the problem.]

3. a. When $P(x) = (x^4 + x^3)$ is divided by $(x + a) = (x - (-a))$, the Remainder Theorem says that the remainder is $P(-a) = (-a)^4 + (-a)^3 = a^4 - a^3$.

b. $x^4 + x^3 = q(x) \cdot (x + a) + (a^4 - a^3)$.

or

$$\frac{x^4 + x^3}{x + a} = q(x) + \frac{a^4 - a^3}{x + a}$$

Part D. Quadratic functions and their applications

1.

$$\begin{aligned} y &= -2x^2 + 8x + 42 \\ y &= -2(x^2 - 4x) + 42 \\ y - 8 &= -2(x^2 - 4x + 4) + 42 \\ y - 8 &= -2(x - 2)^2 + 42 \\ y &= -2(x - 2)^2 + 50 \end{aligned}$$

Vertex is (2, 50).

2. Let x = the length of each of the three horizontal segments.
Then the lengths two vertical segments are $\frac{1}{2}(400 - 3x)$.

The goal is to maximize the area, which is $A(x) = \frac{1}{2}(400 - 3x)\,x$.
Graph this function on your calculator to find that its maximum is at (66.667, 6666.667).
So the maximum area is about 6666.667 square meters.

Other methods: You might have chosen a different length in the diagram as your variable, but in any case, you should have gotten a maximum area of about 6666.667 m^2.