Part A (25%) ______	
Part B (45%) ______	
Part C (30%) ______	
overall ___________	

General instructions: Write a complete, fully explained solution to each problem, except where directions say otherwise. The quality of your responses will be a factor in grading.

Part A. Complex number system

1. Suppose that $w = a + bi$ is a complex number lying in the 4th (lower right) quadrant of the plane. In which quadrant is each of the following complex numbers located? Write a formula or calculation to justify each answer.

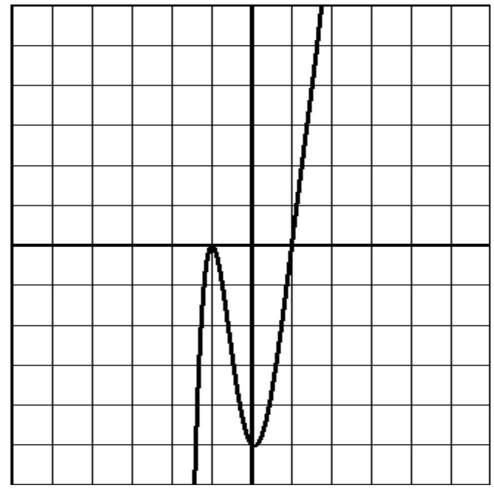
a. $\overline{w}$ (the conjugate of w)

b. $-w$

c. $\frac{1}{w}$

2. For any complex number $z = a + bi$, calculate z^3 and $(\bar{z})^3$, simplifying your answers as much as possible, then reach a conclusion about how the complex numbers z^3 and $(\bar{z})^3$ are related to each other.

Part B. Polynomials



1. You are given the following information about a polynomial $P(x)$:

- $P(x)$ is a degree 5 polynomial with real coefficients.
- The graph of $P(x)$ for real numbers x is given on the grid. Its only x-intercepts are at $x = \pm 1$.
- In the real number system, $P(x)$ factors into 3 linear factors and 1 irreducible quadratic factor.
- In the complex number system, $P(2 - i) = 0$.

Find the factorization of $P(x)$ in the real number system.

2. Without using your calculator, find all solutions to the equation $z^3 = 8$ in the complex number system. Show your complete work.

3. Each of the following arguments reaches an incorrect conclusion. Identify the erroneous step in each argument, identifying which theorem was misused and explaining why the use is incorrect.

a. "Suppose that a rational function $f(x)$ is known to have values $f(2) = -1$ and $f(3) = 4$. Then, $f(x)$ must have a zero in the interval $2 < x < 3$."

b. "Suppose it is given that $C(\mathrm{x})$ is a third-degree polynomial with real coefficients and exactly one x-intercept. By the Fundamental Theorem of Algebra, $C(x)$ must have two non-real complex zeroes. Also, by the Complex Conjugates Theorem, those two non-real zeroes must be conjugates of each other."

Part C. Rational functions

For the problems on this page, use the function $R(x) = \dfrac{3x^3 + 6x^2 - 9x}{x^2 + x}$.

1. Without using a calculator, find the discontinuities of the graph of $R(x)$, and identify the type of each discontinuity (hole, jump, etc.). Justify your answers algebraically.

2. Without using a calculator, solve the inequality $R(x) > 0$. Show your complete reasoning.

3. Find the polynomial $P(x)$ that best approximates the *end behavior* of $R(x)$ as $x \to \infty$.
In other words, find the polynomial $P(x)$ that is an asymptote for $R(x)$ as $x \to \infty$.