Name ____________________________________ Honors Advanced Mathematics Test
December 3, 2004 Chapter 3 Answers

Part A. Using exponent and log properties

1. log(e) and ln(0.1) are opposite reciprocals, because

$$\ln(0.1) = \ln(\tfrac{1}{10}) = \ln(1) - \ln(10) = -\ln(10) = -\frac{\log(10)}{\log(e)} = -\frac{1}{\log(e)}.$$

2. Begin with the known property $a = 10^{\log a}$.

Raise both sides to the power $\frac{x}{\log a}$ to get $a^{\left(\frac{x}{\log a}\right)} = (10^{\log a})^{\left(\frac{x}{\log a}\right)}$.

The power-to-a-power property gives $a^{\left(\frac{x}{\log a}\right)} = 10^{\left(\log a \frac{x}{\log a}\right)}$.

Simplify the exponent to get $a^{\left(\frac{x}{\log a}\right)} = 10^{x}$.

3. $\log_4(1.2) = \log_4(\frac{6}{5}) = \log_4(\frac{2\cdot 3}{5}) = \log_4(2) + \log_4(3) - \log_4(5) = \frac{1}{2} + A - B$.

Part B. Properties and re-expression

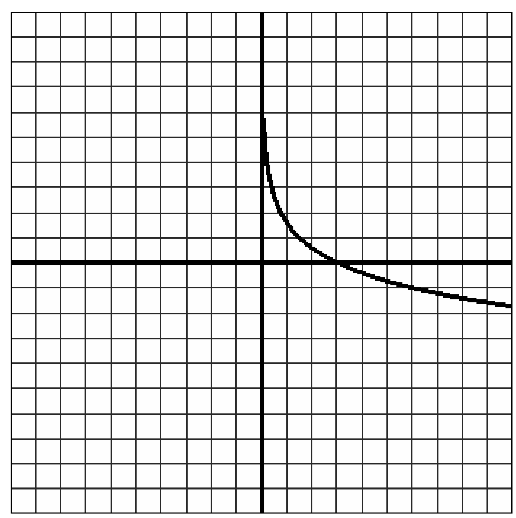
1. No, because $\dfrac{f(x+1)}{f(x)} = \dfrac{2\cdot 3^{x+1}+5}{2\cdot 3^{x}+5}$ which is not a constant, so $f(x+1) \neq c\,f(x)$.

2. **a.** $y = 10^{x/3}$ or $y = (\sqrt[3]{10}\,)^{x}$.

b. $\frac{x}{3} = \log_{10}(y)$ or $x = \log_{\sqrt[3]{10}}(y)$.

3. $\ln y = 0.3 \ln x + 2.4$; $y = e^{2.4}\cdot x^{0.3}$.

Part C. Graphs and transformations



1. $F^{-1}(x) = \log_{1/2}(\frac{x}{3})$ [graph shown at right]

domain of F^{-1} = positive real numbers

range of F^{-1} = all real numbers

2. **a.** $g(x) = \log_7(x) = \dfrac{\log_3(x)}{\log_3(7)} = \dfrac{f(x)}{\log_3(7)}$, so a vertical shrink by $\log_3(7)$ transforms $f(x)$ into $g(x)$.

b. $j(x) = 7^x = (3^{\log_3 7})^x = 3^{(\log_3 7)\cdot x} = h((\log_3 7)\cdot x)$, so a horizontal shrink by $\log_3(7)$ transforms $h(x)$ into $j(x)$.

Part D. Applications

1. $5\text{ kg}\cdot(\frac{1}{2})^{\frac{1000000}{449000000}} = 4.992$ kg.

2. Solve $5000 = 4000\cdot e^{r\cdot 3}$ to get $r = \frac{1}{3}\ln(\frac{5000}{4000}) \approx 0.07438 = 7.438\%$.

3. Model: $P(t) = \dfrac{500}{1+4(0.906574)^{t}}$. Answer: $P(25) \approx 371.90$, so about 372 people.