

### Part A. Trigonometric function definitions and values

- Use  $x = -3$ ,  $y = -\sqrt{40}$ ,  $r = 7$  to get  $\sin(\theta) = -\frac{\sqrt{40}}{7}$ ,  $\tan(\theta) = \frac{\sqrt{40}}{3}$ ,  $\sec(\theta) = -\frac{7}{3}$ .
  - Solve the equation  $\cos x = -\frac{3}{7}$  graphically on your calculator. One way to do this is to graph  $y_1 = \cos x$  and  $y_2 = -\frac{3}{7}$ , and see where they intersect. There are many intersections, but the only intersection in the interval  $[\pi, \frac{3\pi}{2}]$  is at  $x \approx 4.27$ . Answer: 4.27 radians.
- Diagram: see page 360 Figure 4.33. Answers:  $\sin(\frac{5\pi}{3}) = -\frac{\sqrt{3}}{2}$ ,  $\cos(\frac{5\pi}{3}) = \frac{1}{2}$ ,  $\tan(\frac{5\pi}{3}) = -\sqrt{3}$ .
- Represent  $x$  and  $x + \pi$  as angles terminating at opposite points on a circle. (Note that in this diagram, the  $x$  is an angle, not a coordinate.) Since these points have opposite horizontal coordinates and equal radii, the cosine values are opposite. Conclusion:  $\cos(x + \pi) = -\cos(x)$ .

### Part B. Geometric applications

- Area of the circle is  $\pi \cdot 6^2 = 36\pi$ , so this sector is  $\frac{12\pi}{36\pi} = \frac{1}{3}$  of the circle.
  - $\frac{1}{3} \cdot 2\pi = \frac{2}{3}\pi$ .
  - arc length = radius  $\cdot$  angle =  $6 \cdot \frac{2}{3}\pi = 4\pi$ ; sector perimeter =  $6 + 6 + 4\pi = 12 + 4\pi \approx 24.57$ .
- This is a right triangle, so one of the angles is  $90^\circ = \pi/2$  radians.  
Let  $\theta$  be the angle between the 4 and 5 sides. Solve  $\sin \theta = \frac{3}{5}$  graphically:  $\theta \approx 0.644$  radians.  
Use the angle sum of  $180^\circ = \pi$  radians to get the remaining angle.  
Answers: 0.644, 0.927, and  $\pi/2$  ( $\approx 1.571$ ) radians. Or in degrees:  $36.9^\circ$ ,  $53.1^\circ$ ,  $90^\circ$ .
- Use a right triangle whose vertices are the center, the midpoint of a side, and an adjacent vertex. (A set of 16 of these triangles make up the entire octagon.) The central angle is  $2\pi/16 = \pi/8$  (or  $360^\circ/16 = 22.5^\circ$ ), and the opposite side is 3". The hypotenuse of this triangle is the circle's radius  $r$ . Use  $\sin \frac{\pi}{8} = \frac{3}{r}$  (or  $\sin 22.5^\circ = \frac{3}{r}$ ) to get that  $r \approx 7.839$ . Then the circumference of the circle is  $2\pi r \approx 49.256$ .

### Part C. Sinusoidal modeling

- Rewrite in standard form (as on page 374):  $f(x) = -3 \cos(2\pi(x - \frac{-1}{2})) + 5$ .
  - amplitude = 3
  - period = 1
  - vertical translation = 5
  - phase shift =  $-1/2$
- Most common answers:  $y = 3 \cos(\frac{2\pi}{6}(x - 3.5)) - 1$  or  $y = 3 \cos(\frac{2\pi}{6}(x + 2.5)) - 1$ .
- Most common answers:  $h(t) = 102 + 33 \sin(\frac{2\pi}{30}(t - 7.5))$  or  $h(t) = 102 + 33 \sin(\frac{2\pi}{30}(t + 22.5))$ .

### Part D. The other four trigonometric functions

- Diagram: see page 380 Figure 4.49. The terminal radius segment of angle  $\theta$  has a slope of  $\tan \theta$ .
  - Since the radius segments for angles  $\theta$  and  $(\theta + \pi)$  are collinear, they have the same slope. Thus  $\tan(\theta) = \tan(\theta + \pi)$ , so the tangent function has a period of  $\pi$ .
- $\cot(2/3) = 1/\tan(2/3) \approx 1/0.787 \approx 1.271$ .
  - Solve graphically on your calculator (e.g., find intersections of  $y_1 = 1/\tan(x)$  and  $y_2 = 2/3$ ). Some of the infinitely many solutions are  $-2.159$ ,  $0.983$ ,  $4.124$ .
- Diagram: see page 382 Figure 4.56. At  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ ,  $\sec(x)$  is undefined because  $\cos(x) = 0$ .