Name ______________________________ Honors Pre-Calculus Test
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Part A. Proving identities

1. Use a circle diagram with angles θ and –θ drawn in the standard position. Let (x, y) represent the terminal point of angle θ; then the terminal point of angle –θ is $(x, -y)$. This gives: $\tan(-\theta) = \frac{-y}{x} = -\frac{y}{x} = -\tan(\theta)$. ***Conclusion:*** tan(θ) and tan(–θ) are opposites.

2. $\cos(2u) = \cos(u + u) = \cos u \cos u - \sin u \sin u = \cos^2 u - \sin^2 u$.

Next, the Pythagorean Theorem applied to a right triangle with sides cos u, sin u, and 1 gives the Pythagorean identity: $\cos^2 u + \sin^2 u = 1$, so $\cos^2 u = 1 - \sin^2 u$.

Finally, substitute: $\cos(2u) = \cos^2 u - \sin^2 u = (1 - \sin^2 u) - \sin^2 u = 1 - 2\sin^2 u$.

3. Begin with the result of problem **2**, and substitute $u = \frac{x}{2}$, $2u = x$. This gives:

$$\begin{aligned} \cos x &= 1 - 2\sin^2(\tfrac{x}{2}) \\ -1 + \cos x &= -2\sin^2(\tfrac{x}{2}) \\ \tfrac{1-\cos x}{2} &= \sin^2(\tfrac{x}{2}) \\ \pm\sqrt{\tfrac{1-\cos x}{2}} &= \sin(\tfrac{x}{2}) \end{aligned}$$

4. $$\begin{aligned} \tan^4 x + \tan^2 x &= \tan^2 x\ (\tan^2 x + 1) \\ &= \tan^2 x\ \sec^2 x && \text{[by the Pythagorean identity } \tan^2 x + 1 = \sec^2 x] \\ &= (\sec^2 x - 1)\sec^2 x && \text{[by the Pythagorean identity } \tan^2 x + 1 = \sec^2 x] \\ &= \sec^4 x - \sec^2 x. \end{aligned}$$

Part B. Using the identities

1. a. $$\cos(\tfrac{13}{12}\pi) = \cos(\tfrac{1}{3}\pi + \tfrac{3}{4}\pi) = \cos(\tfrac{1}{3}\pi)\cos(\tfrac{3}{4}\pi) - \sin(\tfrac{1}{3}\pi)\sin(\tfrac{3}{4}\pi) = \tfrac{1}{2}\ \tfrac{-\sqrt{2}}{2} - \tfrac{\sqrt{3}}{2}\ \tfrac{\sqrt{2}}{2} = \tfrac{-\sqrt{2}-\sqrt{6}}{4}.$$

b. $$\sin(\tfrac{5}{8}\pi) = \sin(\tfrac{1}{2}\cdot\tfrac{5}{4}\pi) = +\sqrt{\frac{1-\cos(\frac{5}{4}\pi)}{2}} = \sqrt{\frac{1-\frac{-\sqrt{2}}{2}}{2}} = \sqrt{\frac{1}{2}+\frac{\sqrt{2}}{4}}.$$

2. a. $\sin(x + \frac{3}{2}\pi) = \sin x \cos(\frac{3}{2}\pi) + \cos x \sin(\frac{3}{2}\pi) = \sin x \cdot 0 + \cos x \cdot (-1) = -\cos x$.

b. $$\frac{\cos x}{1+\sin x} - \frac{\cos x}{1-\sin x} = \frac{(\cos x)(1-\sin x) - (\cos x)(1+\sin x)}{(1+\sin x)(1-\sin x)} =$$
$$\frac{\cos x - \cos x \sin x - \cos x - \cos x \sin x}{1-\sin^2 x} = \frac{-2\cos x \sin x}{1-\sin^2 x} = \frac{-2\cos x\sin x}{\cos^2 x} = \frac{-2\sin x}{\cos x} = -2\tan x.$$

3. ***<u>If your test had the equation</u>*** <u>$\tan(2x) = -2\sec(2x)$</u>

$\frac{\sin(2x)}{\cos(2x)} = -2\frac{1}{\cos(2x)} \Rightarrow \sin(2x) = -2$ which has no solutions because the range of sine is [–1, 1].

3. ***<u>If your test had the equation</u>*** <u>$\tan(2x) = -\frac{1}{2}$</u> <u>2 $\sec(2x)$</u>

$$\frac{\sin(2x)}{\cos(2x)} = -\frac{1}{2}\frac{1}{\cos(2x)} \Rightarrow \sin(2x) = -\frac{1}{2} \Rightarrow 2x = \frac{7\pi}{6} + 2n\pi \text{ or } 2x = \frac{11\pi}{6} + 2n\pi$$
$$\Rightarrow x = \frac{7\pi}{12} + n\pi \text{ or } x = \frac{11\pi}{12} + n\pi$$

Solutions in the specified interval: $\{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\}$.