Name ______________________________ Honors Advanced Mathematics Test
Class block ____ Sections 6.1–6.3 (in main text)
March 9, 2005

Part A (30%)	______
Part B (30%)	______
Part C (40%)	______
overall	______

Write complete, fully explained solutions. If you use your calculator for a significant step, tell what you did on the calculator.

Part A. Vectors

1. Let **u** = <5, –12> and **v** = <–8, 6>.

a. Find | **u** |.

b. Find a unit vector that has the same direction as **u**.

c. Calculate the dot product **u** · **v**.

d. Find the angle between vectors **u** and **v**.

e. Make up any non-zero vector **w** that is perpendicular to **v**. Justify that **v** ⊥ **w**.

2. An airplane is flying towards on a compass heading (bearing) of 170° at 460 mph. A wind is blowing towards the compass direction 200° at 80 mph.

a. Find the vector that describes the actual flight of the plane as affected by the wind, expressed in component form.

b. Find the actual ground speed and compass direction of the airplane.

Part B. Vector applications to geometry

1. Triangle ABC has A = (2, 2), B = (–5, 4), and C = (–3, 6). Let D be the midpoint of side BC.

a. Find vector $\vec{AD}$ in component form.

b. Using a vector method, determine whether $\vec{AD}$ and $\vec{BC}$ are perpendicular.

c. Using a vector method, find the coordinates of the centroid of the triangle. (The centroid is the point where the medians intersect.)

2. In this problem you will be asked to prove the following theorem using a vector method.

Theorem: If a parallelogram is a rhombus, then it has perpendicular diagonals.

a. Draw a parallelogram. Let **u** and **v** be the vectors representing two adjacent sides. Identify the diagonals of the parallelogram, in terms of **u** and **v**.

b. Write what is given and what needs to be proved in the theorem statement. Use vector operations expressed in terms of the vector names set up in part **a**.

Given:

To prove:

c. Now write a vector proof of the theorem.

Part C. Parametric equations

1. Suppose that the motion of an object along a line L is described by this vector equation.

$(x, y) = (3, -2) + t \langle 4, 5 \rangle$.

a. Write a pair of parametric equations that is equivalent to the given vector equation.

$x =$

$y =$

b. Write a different pair of parametric equations that also describes motion along line L, but moving *twice as fast* as in the given equations.

c. Write another pair of parametric equations that describes motion along a line that is parallel to, but not the same as, line L.

2. Given: parametric equations $x = 4 \sin t$ and $y = 4 \cos t$. (Note which equation has the sine and which equation has the cosine.)

a. Find the (x, y) coordinates for each of these t-values.

$t = 0$:

$t = \pi/2$:

$t = \pi$:

$t = 3\pi/2$:

b. Describe the shape of the graph and the direction of movement along the graph. (Justification not required.)

c. Find a restriction (interval of t-values) such that you would only get the portion of the graph lying in the 4th quadrant (lower-right quadrant).

3. The following parametric equations describe the flight of a model rocket:

$x = 8t$ (in feet)

$y = -16t^2 + 80t + 3$ (in feet)

Time t is measured in seconds, and the rocket was launched at time $t = 0$.

a. At what angle was the rocket launched?

b. What was the speed of the rocket when launched?

c. At what time and what location does the rocket hit the ground?