Scoring: Problems **1–4** count 10% each; **5–8** count 15% each.

General instructions: Show or briefly explain how you get your answers. Where applicable, state answers first in terms of counting operations ($n!$, ${}_nP_r$, ${}_nC_r$) **and then** as ordinary numbers. For example, first write ${}_6C_2$, then write 15. Probability answers may be rounded to 3 decimal places.

1. Mr. Kresser (who would know) says that Indiana license plates are labeled as follows:

- a number from 1 to 92 representing the 92 counties in the state
- a letter from A to Z
- four digits, each from 0 to 9

Here is an example of such a license plate: 87A 0842

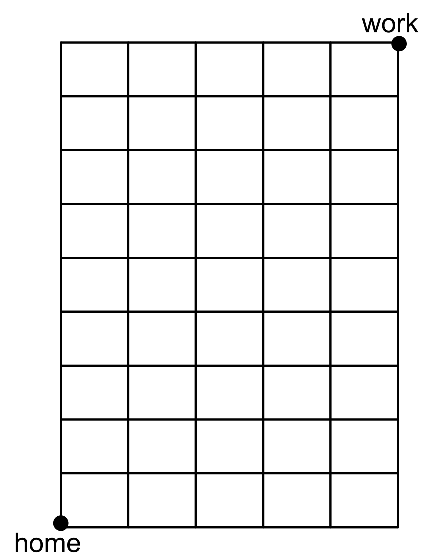
a. How many different Indiana license plates are possible?

b. In what percentage of these license plates would the last four digits be all different?

2. A city resident wants to travel from home (0, 0) to work (5, 9). The only legal steps are to move right or up along the gridlines.

a. How many paths exist from home to work?

b. On the way to work, this person needs to stop at a bakery located at (2, 6). Of all the paths that can be taken from home to work, what percentage of them go through the bakery?



3. **a.** Show a reasonably practical way to find the value of ${}_{300}C_3$ without using your calculator.

b. In the expansion of $(4x - y)^{12}$, find the coefficient of the x^5y^7 term.

4. A coin is bent so that its outcomes are unfair: it shows heads 55% of the time, tails 45% of the time. Suppose that this coin is flipped 5 times.

a. For the unfair coin described above, find the probability that the coin comes up tails **at least** 4 of the 5 times.

b. Here is a game of chance based on flipping the unfair coin 5 times.

- If tails comes up exactly 3 times, the player wins \$3.
- If tails comes up more than 3 times, the player wins \$10.
- If tails comes up less than 3 times, the player loses \$5.

Calculate the expected value of this game, and tell whether this game is advantageous or disadvantageous to the player.

5. A card game is played using a special set of 8 cards. On each card a number is printed in some color. Here are the colors and numbers for the 8 cards:

red 2, green 2, blue 3, red 4, red 5, red 6, blue 6, green 8

One card is drawn at random from this special set. Let R be the event of drawing a card with a red number, and let E be the event of drawing a card with an even number.

a. Find $P(\text{R and E})$.

b. Find the conditional probability $P(\text{R} \mid \text{E})$.

c. Are events R and E independent? Justify your answer.

d. Find the probability of event R given that event E does **not** occur.

6. **a.** A box contains 7 green tennis balls and 3 yellow tennis balls. Suppose that two tennis balls are taken from the box at the same time. Find the probability that they have different colors.

b. A box contains 7 green tennis balls and 3 yellow tennis balls. Suppose that a tennis ball is drawn from the box, then replaced. Then, a tennis ball is drawn from the box again. Find the probability that the colors observed in the two draws were different.

c. A box contains 7 green tennis balls and 3 yellow tennis balls. If you start drawing tennis balls one-at-a-time without replacement, what is the probability that the **first** yellow ball will appear on the **third** draw?

7. On a particular school day, 20% of all students were sick. Of those students who were sick, 1/2 of them missed school. Of those who were not sick, 1/16 of them missed school.

a. What is the probability that a student was sick and missed school?

b. What percent of the students were present on this day of school?

c. A student missed school on this day. What is the probability that this student was sick?

8. From one day to the next, a person with the flu has an 80% chance of having the flu the next day. Otherwise, they will have recovered from the flu and become immune. If someone does not have the flu, they have a 15% chance of getting the flu the next day.

Suppose that on February 1, 70% of students were healthy, 20% had the flu, and 10% had already had the flu and were immune.

a. For February 2, calculate the percent of students who haven't had the flu, who currently have the flu, and who were already immune on February 2. Use an equation that includes a transition matrix.

b. The maximum percent of students having the flu will occur on what day in February? Show or tell how you get your answer. You may use your calculator.

c. What is the steady-state matrix for this model? Explain why.