

Name _____
Class block _____
June 13, 2005

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|--------------|-------|
| Part A (25%) | _____ |
| Part B (25%) | _____ |
| Part C (25%) | _____ |
| Part D (25%) | _____ |
| overall | _____ |

Part A. Sequences

- Here is a recursive definition of a sequence: $t_1 = 6$; $t_n = \frac{1}{4} t_{n-1}$ for $n \geq 2$.
Write an explicit formula for t_n describing the same sequence.

- Let D_n represent the number of diagonals that can be drawn in a convex n -sided polygon. For example, $D_4 = 2$, because quadrilaterals have two diagonals.
 - Write a recursive formula for D_n .

 - Write an explicit formula for D_n .

- Find an arithmetic sequence in which no term is divisible by 2, 3, or 5.
 - List the first five terms of your sequence.

 - Write an explicit formula for the n th term of your sequence.

Name _____

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Sequences, Series, and Limits (9.4, 10.3)

page 2

Part B. Evaluating series

Directions: Find the value of each series. Simplify your answers as much as possible. If a series does not converge to a value, write “The series diverges.”

answers

1. $\sum_{k=3}^{100} (7k + 5)$

1. _____

2. $\sum_{n=1}^{\infty} \left[4 \cdot \left(-\frac{1}{3}\right)^n \right]$

2. _____

3. $\sum_{r=1}^{1000} t_r$ whose terms are defined recursively as follows:

3. _____

$$t_1 = 5;$$

$$t_r = t_{r-1} - 2.5, \text{ for } 2 \leq r \leq 1000.$$

4. $16 - 4i - 1 + \frac{1}{4}i + \frac{1}{16} + \dots$

4. _____

(where i is the imaginary unit of the complex numbers)

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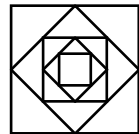
page 3

Part C. More series and their applications

1. Find the sum of the series $2 - 4 + 6 - 8 + 10 - 12 + \dots + 998 - 1000$.

2. If you go back through 20 generations, how many ancestors do you have? (Count your 2 parents as the first generation back, 4 grandparents as the second generation back, and so on. Disregard the possibility of duplications.)

3. The figure at the right shows the first five of an infinite sequence of squares. The outermost square measures 3 inches by 3 inches, and each of the other squares is obtained by joining the midpoints of the sides of the square before it. Find the sum of the perimeters of all the squares in the infinite sequence.



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Part D. Limits

1. Find $\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2}$ using an algebraic method. Show your work.

2. Find the exact values of these limits by any method. You do not need to show work.

a. $\lim_{n \rightarrow \infty} \frac{4 + 5n^2}{3n + 7n^2}$

b. $\lim_{x \rightarrow 0} \frac{2x}{\tan x}$

3. On the grid, sketch a graph of a function $F(x)$ having the following properties:

$$F(-2) = 3$$

$$\lim_{x \rightarrow (-2)^+} F(x) = 5$$

$$\lim_{x \rightarrow (-2)^-} F(x) = 7$$

$$\lim_{x \rightarrow 1} F(x) = \infty$$

$$\lim_{x \rightarrow \infty} F(x) = -\infty$$

