Name ______________________________ Honors Advanced Mathematics Test

June 13, 2005 Sequences, Series, and Limits (9.4, 10.3) Answers

Part A. Sequences

1. Geometric sequence $t_n = 6 \cdot (\frac{1}{4})^{n-1}$.

2. **a.** $D_3 = 0$; $D_n = D_{n-1} + (n-2)$ for all $n \geq 4$.

b. $D_n = \dfrac{n(n-3)}{2}$ for all $n \geq 3$ (because there are $(n-3)$ diagonals from each of the n vertices).

3. ***One of many possible answers:*** 1, 31, 61, 91, 121,…; $t_n = 1 + 30(n-1)$.

Part B. Evaluating series

1. finite arithmetic series: $\dfrac{98(26+705)}{2} = 35819$

2. infinite geometric series: $\dfrac{-\frac{4}{3}}{1-(-\frac{1}{3})} = -1$

3. finite arithmetic series: $\dfrac{1000(5+(-2492.5))}{2} = -1243750$

4. infinite geometric series with ratio $-\frac{1}{4}i$; its sum is $\dfrac{16}{1-(-\frac{1}{4}i)} = \dfrac{16}{1+\frac{1}{4}i} = \frac{256}{17} - \frac{64}{17}i$

Part C. More series and their applications

1. Easiest to add positive and negative terms separately.

$$2 + 6 + \cdots + 998 = \frac{250(2+998)}{2} = 125000$$

$$-4 - 8 - \cdots - 1000 = \frac{250(-4-1000)}{2} = 125500$$

Sum of whole series = –500.

2. finite geometric series: $2 + 4 + \cdots + 2^{20} = \dfrac{2(1-2^{20})}{(1-2)} = 2^{21} - 2 = 2097150$ ancestors.

3. infinite geometric series: $12 + 6\sqrt{2} + 6 + 3\sqrt{2} + \cdots = \dfrac{12}{1-\frac{1}{\sqrt{2}}} = \dfrac{12\sqrt{2}}{\sqrt{2}-1} \approx 40.971$ sq. in.

Part D. Limits

1. $\lim\limits_{x\to 2} \dfrac{x^3-2^3}{x-2} = \lim\limits_{x\to 2} \dfrac{(x-2)(x^2+2x+4)}{x-2} = \lim\limits_{x\to 2}(x^2+2x+4) = 2^2 + 2\cdot 2 + 4 = 12.$

2. **a.** $\frac{5}{7}$ **b.** 2 (use table or graph)

3. Graph must have these features:

- Isolated point at (–2, 3)
- Hole at (–2, 5) approached from the right
- Hole at (–2, 7) approached from the left
- Vertical asymptote at $x = 1$, with graph rising towards the asymptote on both sides
- Right end behavior in the downward direction