Name ______________________________ Honors Pre-Calculus Test Answers
October 11, 2001 Chapter 1, emphasizing 1.4–1.6

Part A. Functions and their inverses

1. There must be only one *x*-value corresponding to each *y*-value. Graphically, the graph must pass the horizontal line test.

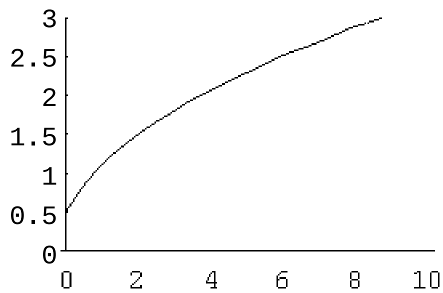
2. Exchange the *x* and *y* variables, then solve for *y*. Answer: $f^{-1}(x) = \dfrac{x+3}{x-2}$.

Domain of f^{-1} = Range of f = all real numbers except 2.
Range of f^{-1} = Domain of f = all real numbers except 1.

3. One of many possible answers: $x = t^2$, $y = t$, Tmin = –10, Tmax = 10.

Part B. Combining functions

1.



2. **a.** $(f \circ g)(5) = f(g(5)) \approx f(-1.3)$, which is undefined because –1.3 is outside the domain of *f*.

b. $(g \circ f)(5) = g(f(5)) \approx g(2.2) \approx 1.3$.

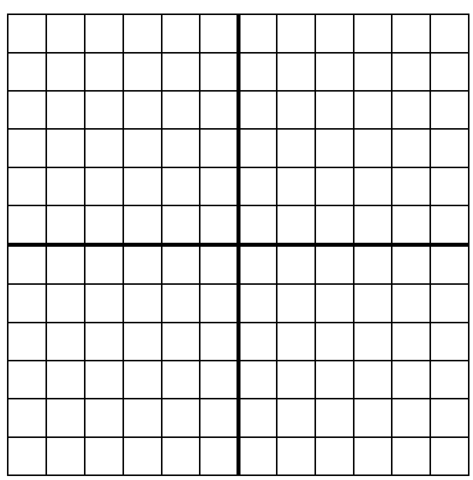
3. The functions are inverses if and only if $h(j(x)) = x$ and $j(h(x)) = x$.

$h(j(x)) = h(\sqrt[3]{x+1}) = \left(\sqrt[3]{x+1}\right)^3 - 1 = x - 1 + 1 = x.$

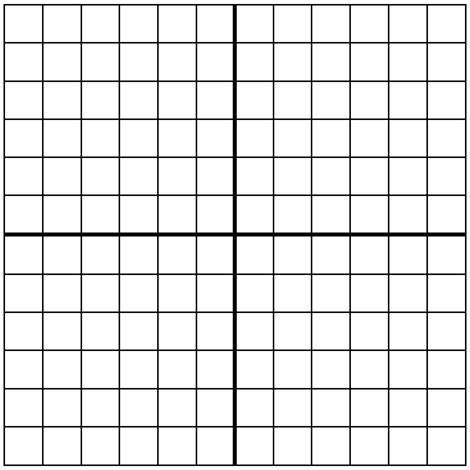
$j(h(x)) = j(x^3 - 1) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x.$

Part C. Transformations

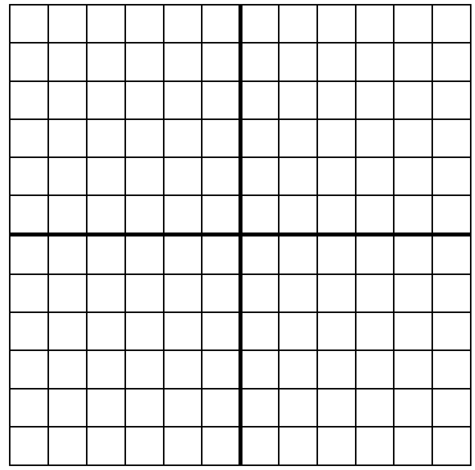
1. Graph $\frac{1}{2}F(-x)$.



2. Graph $F(x + 3) - 1$.



3. Graph $F(2x) + 1$.



4. Translate down by 4, then translate left by 1, then stretch vertically by a factor of 5.

Part D. Modeling with functions (D block on Oct. 11)

1. Let x represent the lengths of two opposite sides. The length of the third size is $(2000 - 2x)$. The area function $A(x) = x(2000 - 2x)$ has its maximum at $A(500) = 500000$.

OR: Let x represent the length of the side opposite the river. The other two sides have lengths $(2000 - x)/2$. The area function $A(x) = x((2000 - x)/2)$ has its maximum at $A(1000) = 500000$.

Conclusion from either approach: The optimal dimensions are 500 by 1000 feet (with two 500' sides and one 1000' side), providing an area of 500000 square feet.

2. Let A represent the area of the circle, s the side length of the square. Use a 45°-45°-90° triangle to get that the radius of the circle is $\frac{s}{\sqrt{2}}$. Then $A = \pi(\frac{s}{\sqrt{2}})^2$, which simplifies to $A = \pi s^2 / 2$. Solve for s to get $s = \sqrt{\frac{2A}{\pi}}$.

Part D. Modeling with functions (G block on Oct. 12)

1. Let p represent the perimeter, and let A represent the area. The length of a side $p/3$; the length of half of a side is $p/6$. Use a 30°-60°-90° triangle to get that the height is $\frac{p}{6}\sqrt{3}$. Then the area is $A = \frac{1}{2} \cdot \frac{p}{3} \cdot \frac{p}{6}\sqrt{3} = \frac{\sqrt{3}}{36} p^2$.

2. Let x represent the length of each of the three vertical segments. Then the two horizontal segments have lengths $\frac{500-3x}{2}$. The area function $A(x) = x \cdot \frac{500-3x}{2}$ has its maximum at $A(83.33333) \approx 10416.67$.

OR: Let x represent the length of each of the two horizontal segments. Then the three vertical segments have lengths of $\frac{500-2x}{3}$. The area function $A(x) = x \cdot \frac{500-2x}{3}$ has its maximum at $A(125) \approx 10416.67$.

Conclusion from either approach: The optimal design consists of three vertical segments of about 83.33' each, and two horizontal segments of 125' each. The area enclosed is about 10416.67 square feet.