Name ___________________________ Honors Pre-Calculus Test Answers
November 29, 2001 Chapter 3

Part A. Properties of logarithmic functions

1. Let $x = \ln a$. Then $e^x = a$. Take both sides to the nth power to get $(e^x)^n = a^n$.
Use the exponential power property to rewrite as $e^{nx} = a^n$.
Now rewrite as a logarithmic equation: $\ln (a^n) = nx$. Substitute to get $\ln (a^n) = n \ln a$.

2. Let $y = \log_b x$. Then $x = b^y$.
Take ln of both sides to get $\ln x = \ln (b^y)$.
Use the logarithm power property to rewrite as $\ln x = y \ln b$.
Finally, divide to get $y = \frac{\ln x}{\ln b}$ and substitute to get $\log_b x = \frac{\ln x}{\ln b}$.

3. **a.** $\ln 7 = \log_e 7 = \log 7 / \log e$, so calculate LOG(7)/LOG(e).

b. ln 7 is the solution to the equation $e^x = 7$, so you can do either of the following:

- Graph $y = e^x$ and $y = 7$, and find their intersection.
- Graph $y = e^x - 7$ and find its zero.

Part B. Growth and decay models

1. Compounded monthly: \$1,111,291.04
Compounded continuously: \$1,112,770.46
Difference: \$1479.42

2. **a.** Solve $250 = 500\, e^{-0.0001216\, t}$ to get $t \approx 5700$ years.

b. Solve $1 = 500\, e^{-0.0001216\, t}$ to get $t \approx 51107$ years.

3. A population is modeled using a logistic function that satisfies these conditions:

- initial population = 1000
- population after 5 years = 1300
- maximum sustainable population = 2000

3 **a.** Graph should be shaped like those on page 268, and approach a horizontal asymptote of 2000.

b. Begin with the general form $f(x) = \dfrac{c}{1 + a \cdot b^x}$. In this form, the maximum population is c, giving $f(x) = \dfrac{2000}{1 + a \cdot b^x}$. Substitute the point $f(0) = 1000$ to get $a = 1$. Now we have $f(x) = \dfrac{2000}{1 + b^x}$. Finally, substitute $f(5) = 1300$ to get $b \approx 0.8835496$. Final answer:
$f(x) = \dfrac{2000}{1 + 0.8835496^x}$.

c. Population ≈ 1551.

Part C. Exponential functions and equations

Note: Problem **1** was a short-answer question with no partial credit.

1. Translate left by 3 units, then shrink horizontally by a factor of 2.

2. Rewrite as an exponential equation $3x = \log_2 e$. Then $x = (\log_2 e) / 3$.

3. $f(x) = 100 \cdot 4^x = 100 \cdot (e^{\ln 4})^x = 100 \cdot e^{(\ln 4)x}$, or $\approx 100 \cdot e^{1.38629\, x}$.