Name ________________________________ Honors Pre-Calculus Test Answers

December 14, 2001 Sections 4.1–4.4

A1. The pendulum path is an arc of a circle. The arc length is $\frac{2^\circ}{360^\circ}\cdot(2\pi\cdot 52)\approx 1.815$ feet.

A2. $\dfrac{2\pi\text{ rad.}}{1\text{ rev.}}\cdot\dfrac{1\text{ rev.}}{28\pi\text{ in.}}\cdot\dfrac{12\text{ in.}}{1\text{ ft.}}\cdot\dfrac{5280\text{ ft.}}{1\text{ mile}}\cdot\dfrac{55\text{ miles}}{1\text{ hour}}\cdot\dfrac{1\text{ hr.}}{60\text{ min.}}\cdot\dfrac{1\text{ min.}}{60\text{ sec.}}\approx 69.14\dfrac{\text{rad.}}{\text{sec.}}$.

(Here the 28π is the circumference of the tire and $\frac{55\text{ miles}}{1\text{ hour}}$ is the given speed. All of the other factors just equal 1, and are used for the purpose of converting between units.)

A3. Sector area = (fraction of the circle) · (area of the whole circle)

$= \dfrac{\theta}{2\pi}\cdot\pi r^2$ — θ represents the angle of the sector, in radians

$= \frac{1}{2}\theta r^2$

$= \frac{1}{2}\left(\frac{s}{r}\right) r^2$ — because $s = r\theta$

$= \frac{1}{2}sr$.

Here is another approach, in which the fraction is found using the arc length.

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$= \dfrac{s}{2\pi r}\cdot\pi r^2$

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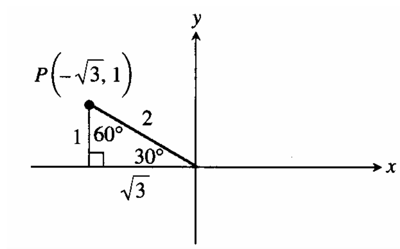

B1. Use the reference triangle shown: $\sin(\frac{5\pi}{6})=\sin(150^\circ)=\sin(30^\circ)=\frac{1}{2}$.

B2. a. $\cos\theta=\frac{x}{r}=\frac{6}{10}=0.6$. $\csc\theta=\frac{r}{y}=\frac{10}{-8}=-1.25$.

b. Solving $\cos\theta = 0.6$ graphically on a calculator (by finding intersections between the cosine graph and the constant 0.6 graph), you get two sets of solutions: $0.927 + 2\pi n$ and $5.356 + 2\pi n$. However, $\theta = 0.927$ and its coterminal angles all fall in the first quadrant, while the given point was in the fourth quadrant. So the only solutions are $5.356 + 2\pi n$ (where n denotes any integer).

B3. $\cot\theta = 0$ means that $\frac{x}{y}=0$, so $x = 0$, meaning that the terminal point of the angle must lie on the y-axis (i.e., at either the top or the bottom of the circle). Now, $\sin\theta < 0$ means that $\frac{y}{r}<0$, so $y < 0$, meaning that the terminal point of the angle must lie in the 3rd or 4th quadrant (i.e., in the lower half of the circle). The only angle fitting both requirements is at the bottom of the circle, $\theta = \frac{3\pi}{2}$.

B4. $(\sin\theta)^2 + (\cos\theta)^2 = 1$. ***Proof:*** $(\sin\theta)^2 + (\cos\theta)^2 = \left(\frac{y}{r}\right)^2+\left(\frac{x}{r}\right)^2=\dfrac{y^2+x^2}{r^2}=\dfrac{r^2}{r^2}=1$.

The substitution $y^2 + x^2 = r^2$ is due to the Pythagorean Theorem on the usual x, y, r triangle.

C1. <u>Shrink vertically by a factor of 5</u> to get $y = \cos(3\theta)$, then <u>translate left by 2</u> to get $y = \cos(3(\theta + 2)) = \cos(3\theta + 6)$. (You could also translate first and shrink second.)

Note that translation by 6 is incorrect; that would give $y = \cos(3(\theta + 6))$.

C2. a. 8 hours (occurring at the point (6,8), among others)

b. Southern Hemisphere, because the shortest day is in June, not December.

c. $12 + \cdots$: average length of a day is 12 hours $\frac{2\pi}{12}$: 12 months in a yearly cycle

C3. a. M is the maximum displacement of the pendulum. In other words, it is the horizontal distance that the pendulum swings to each side from its center resting position.

b. $\text{period}=\dfrac{2\pi}{b}=\dfrac{2\pi}{\sqrt{\frac{32}{52}}}\approx 8.01\text{ sec}.$