Name ___________________________________ Honors Pre-Calculus Test Answers

January 11, 2002 Chapter 4

Part A. Sinusoidal functions and periodicity

1. Note that the average height is 30 inches, the amplitude is 6 inches, and the period is 4 seconds. The function must have a minimum value at $t = 0$, which can be achieved through choices of sin vs. cos and + vs. –, or through a phase shift.

Two possible answers: $h(t) = -6\cos(\frac{2\pi}{4}t) + 30$ or $h(t) = 6\sin(\frac{2\pi}{4}(t-1)) + 30$.

2. Yes. The two terms have fundamental periods of $\frac{\pi}{2}$ and $\frac{\pi}{3}$. The least common multiple of these numbers is π, so both terms have π as a period. Thus the period of their sum $f(x)$ is π.

Part B. Values of trig and inverse trig functions

1. Transform $\sec\theta = -2$ into $\cos\theta = -\frac{1}{2}$. Angles with negative cosines must lie in the 2nd or 3rd quadrant. Since a special triangle shows that $\cos\frac{\pi}{3} = +\frac{1}{2}$, the 1st quadrant reference angle must be $\frac{\pi}{3}$. The corresponding angles in the other quadrants are $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$. Including coterminal angles gives the complete set of answers: $\frac{2\pi}{3} + 2\pi n$ and $\frac{4\pi}{3} + 2\pi n$.

2. θ must be a first quadrant angle. The point (2, 1) is located on the terminal ray, but at a radius of $\sqrt{5}$. Adjust the coordinates to maintain the same angle but achieve the desired radius of 1. *Answer:* $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$.

3. From the given inverse statement, we know that $\cos 130° = w$. This means that w is a negative number. The other quadrant where cosines are negative is the 3rd quadrant. The corresponding angle in that quadrant is 230°, so $\cos 230° = w$ also. *Answers:* 130° and 230°.

Suppose $\cos^{-1} w = 130°$. Find two different angles, in the interval from 0° to 360°, whose cosines equal w.

Part C. Application problems

1. Solve $\tan\theta = \frac{50}{80}$ to get $\theta \approx 0.559$ radians or 32.01°

2. **a.** The minimum and maximum values of $h(t)$ are 5 feet and 75 feet, so the diameter is 70 feet, and the circumference is 70π feet ≈ 219.91 feet.

b. The period of the $h(t)$ is 40 seconds, so that is the amount of time needed to travel the full circumference of the ferris wheel. In just 4 seconds, you would travel $\frac{1}{10}$ of the circumference, which is 7π feet ≈ 21.99 feet.

Part D. More on trig and inverse trig functions

1. The asymptotes of tan(x) are located at $x = \frac{\pi}{2} + n\pi$. The factor of 3 causes a horizontal shrink by a factor of 3, so the asymptotes of tan(3x) are located at $x = \frac{\pi}{6} + n\frac{\pi}{3}$.

2. The cosine function is not one-to-one within the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. For example, both $\cos(-\frac{\pi}{3}) = \frac{1}{2}$ and $\cos(\frac{\pi}{3}) = \frac{1}{2}$, so it would not be possible to define a unique inverse value for $\frac{1}{2}$.

3. $\sec(\tan^{-1} x) = \sqrt{1 + x^2}$. See page 400 Exploration 1 for the triangle that proves this.