Five problems count for 20% each. Write complete, fully explained solutions.

1. Prove the identity $\dfrac{\sin x}{\sec^2 x - 1} = \dfrac{1 - \sin^2 x}{\sin x}$. Present your proof as a chain of equal expressions that begins with $\dfrac{\sin x}{\sec^2 x - 1}$ and ends with $\dfrac{1 - \sin^2 x}{\sin x}$. Justify each step.

2. Without your calculator, find all the zeroes of the function $F(x) = \sin 2x - \sin x$ in the interval $0 \le x < 2\pi$. Show all computational steps, and give exact answers in radians.

3. Beginning from the fact $\cos(2u) = \cos^2 u - \sin^2 u$, derive the cosine half-angle formula.

4. Without your calculator, determine the exact value of cos(165°), using the fact that $165° = \frac{1}{2} \cdot 330°$.

5. Without your calculator, determine the exact value of cos(165°), using the fact that $165° = 120° + 45°$.

Use your calculator to check that your answers to **4** and **5** agree with each other even though they look different.