Name _____ Honors Pre-Calculus Half-Test Answers
January 28, 2002 Sections 5.1–5.4

1. $\dfrac{\sin x}{\sec^2 x - 1} = \dfrac{\sin x}{\tan^2 x}$ (because $\tan^2 x + 1 = \sec^2 x$)

$= \dfrac{\sin x}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

$= \dfrac{\cos^2 x}{\sin x}$

$= \dfrac{1 - \sin^2 x}{\sin x}$ (because $\sin^2 x + \cos^2 x = 1$)

2. $\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x (2 \cos x - 1) = 0$

$\sin x = 0$ or $(2 \cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = 0$ or $x = \pi$ or $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

3. $\cos(2u) = \cos^2 u - \sin^2 u$

$\cos(2u) = \cos^2 u - (1 - \cos^2 u)$

$\cos(2u) = 2 \cos^2 u - 1$

Now let $x = 2u$.

$\cos x = 2 \cos^2(\frac{x}{2}) - 1$

$1 + \cos x = 2 \cos^2(\frac{x}{2})$

$\dfrac{1 + \cos x}{2} = \cos^2(\frac{x}{2})$

$\pm \sqrt{\dfrac{1 + \cos x}{2}} = \cos(\frac{x}{2})$

4. $\cos(165°) = \cos(\frac{1}{2} \cdot 330°) = -\sqrt{\dfrac{1 + \cos 330°}{2}} = -\sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\dfrac{1}{2} + \dfrac{\sqrt{3}}{4}}$.

Note that the ± is chosen to be –, because cos(165°) must be negative.

5. $\cos(165°) = \cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45° = \dfrac{-1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{-\sqrt{2} - \sqrt{6}}{4}$.

Check answers to 4 and 5: Both are ≈ –0.9659.