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April 26, 2002

Honors Pre-Calculus Test
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This test has four multi-part problems, each counting for 25% of the grade.

All of the equations on this test produce non-degenerate curves.

1. A hyperbola has the following properties:

- Vertices at $(-3, 4)$ and $(-3, 0)$.
- Foci at $(-3, 6)$ and $(-3, -2)$.

a. Sketch a graph of this hyperbola.

b. Find an equation describing this hyperbola.

c. Find the slopes of the asymptotes. Then, write equations for the asymptotes.

d. A hyperbola's eccentricity is given by $e = \frac{c}{a}$. Find the eccentricity of this hyperbola.

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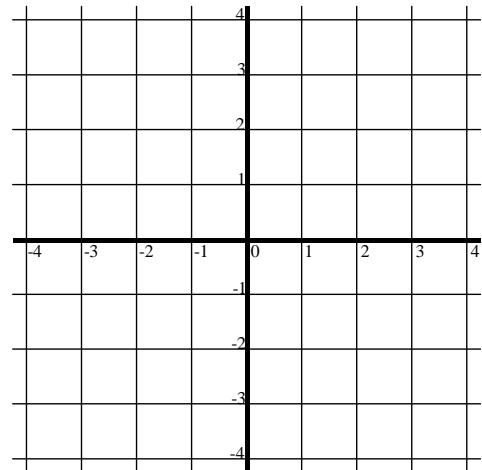
2. a. The equation $x^2 + 4xy + 4y^2 = 1 + x$ describes what type of curve? Show how you get your answer.

b. Give a general definition, in terms of distances, for all curves of the type you named in part a. Illustrate your definition with a picture.

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c. Transform $x^2 + 4xy + 4y^2 = 1 + x$ into a form that could be entered into your calculator.

d. Graph $x^2 + 4xy + 4y^2 = 1 + x$ on this grid.



e. Depending on its type, the curve you graphed will have some of the features listed below. Circle the names of the features that apply to this curve. Then, sketch those features on your graph (in their approximate locations; you don't have to calculate the exact locations of these points and lines).

one focus
two foci

one vertex
two vertices
four vertices

one directrix
two asymptotes

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3. The planet Pluto has an elliptical orbit with the sun at one focus. The distance from Pluto to the sun varies between a minimum of 2763 million miles and a maximum of 4587 million miles. As you answer these questions, you may use the letter M as an abbreviation for *million miles*, and you may round off any distances to the nearest 1 M .

a. Find the eccentricity of Pluto's orbit.

b. Find the lengths of the ellipse's major and minor axes.

c. Suppose this ellipse is placed in a coordinate system, with the sun at $(0,0)$, and with **more than half** of the ellipse's major axis lying along the **positive y-axis**.

On the given axes, sketch a graph of the ellipse.
Make it reasonably close to the correct shape.

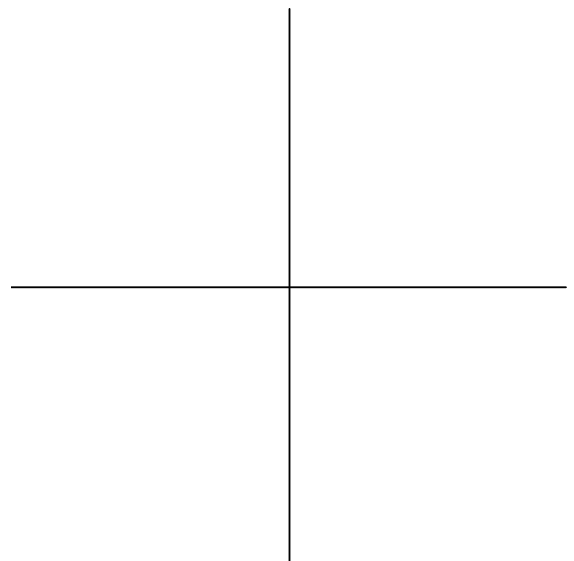
d. Identify the coordinates of the following points.

endpoints of major axis: _____

endpoints of minor axis: _____

foci: $(0, 0)$ and _____

e. Placed in the coordinate system as described above, write an equation for the ellipse.



f. In any ellipse, the sum of the two focal lengths is a constant number. For this ellipse, find that constant number.

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4. Consider the curve described by $2x^2 + 4\sqrt{3}xy - 2y^2 = 16$.
- What type of curve is this? Show how you get your answer.
 - Using the formula $\cot(2\alpha) = \frac{A-C}{B}$, find an angle α that the curve could be rotated so that the axes would be oriented horizontally and vertically.
 - Rotate the curve by making the following substitutions: replace x with $(x \cos \alpha - y \sin \alpha)$ and replace y with $(x \sin \alpha + y \cos \alpha)$. Simplify as much as possible.
 - Find the vertex or vertices of the new curve you found in part c.
 - Find the vertex or vertices of the original curve $2x^2 + 4\sqrt{3}xy - 2y^2 = 16$. (To choose the correct direction of rotation, it may help to look at both graphs on your calculator.)