Name ______________________________ Honors Pre-Calculus Test Answers
October 29, 2002 Sections 2.1–2.4

1. Use the strategy of expressing the rectangle area as a function of a variable, then finding the maximum of this function. For example, let x represent the length, w the width, and $A(x)$ the area as a function of the length. Since $2x + 2w = 25$, $w = \frac{25-2x}{2}$, which gives the area $A(x) = x \cdot w = x \cdot \left(\frac{25-2x}{2}\right)$. If you graph $A(x)$ on your calculator and find the maximum, you will find it occurs where $A(x) = 39.0625$ and $x = 6.25$, so $w = \frac{25-2(6.25)}{2} = 6.25$. Thus the rectangle with perimeter 25 that has maximum area is actually a square, 6.25 units by 6.25 units.

2. First transform the function into standard form, then use the method of completing the square to put it into vertex form (see below). This shows that the vertex is $\left(\frac{7}{2}, \frac{27}{4}\right)$, or (3.5, 6.75).

$$y = -3(x-2)(x-5) = -3(x^2 - 7x + 10) = -3(x^2 - 7x) - 30 = -3\left(x^2 - 7x + \left(\tfrac{7}{2}\right)^2 - \left(\tfrac{7}{2}\right)^2\right) - 30$$
$$= -3\left(x - \tfrac{7}{2}\right)^2 + 3\left(\tfrac{7}{2}\right)^2 - 30 = -3\left(x - \tfrac{7}{2}\right)^2 + \tfrac{27}{4}.$$

3. It's given that $Q(x)$ has zeros at $x = -2$, 1, and 3, and from the graph we see that the zero at -2 has multiplicity 2 since the graph is tangent to the axis. We can apply the Factor Theorem and write a general equation for $Q(x)$ in factored form, with an unknown leading coefficient:

$$Q(x) = a(x+2)^2(x-1)(x-3).$$

Now, if we substitute the other given point (0, 8), we can evaluate a:

$8 = a(0 + 2)^2 (0 - 1) (0 - 3)$

$a = 2/3$

Thus the final answer is $Q(x) = \frac{2}{3}(x+2)^2(x-1)(x-3)$.

4. To prove that the graphs of $C(x)$ and $L(x)$ intersect, we must show that there is a solution to the equation $C(x) = L(x)$, which is equivalent to $C(x) - L(x) = 0$. The difference of a cubic function and a linear function will be another cubic function. So $C(x) - L(x)$ is cubic, and we know that every cubic (in fact, every odd degree polynomial) must have a zero, due to its end behavior along with the Intermediate Value Theorem. Thus, $C(x) - L(x) = 0$ must have a solution, which means that the graphs of $C(x)$ and $L(x)$ must intersect.

5. **a.**

$$\begin{array}{r|l} & 2x - 5 \\ \hline x^2 - 4 & 2x^3 - 5x^2 \qquad + 6 \\ & 2x^3 \qquad - 8x \\ \hline & -5x^2 + 8x + 6 \\ & -5x^2 \qquad + 20 \\ \hline & 8x - 14 \end{array}$$

b. $2x^3 - 5x^2 + 6 = (x^2 - 4)\cdot(2x - 5) + (8x - 14)$

6. The remainder equals $P(c)$. This is the Remainder Theorem. See top of page 201 for proof.

7. The tricky thing here is that $f(x)$ has two zeroes close together, at $x \approx 1.328$ and $x \approx 1.333$. You have to zoom in close to see what's going on. Try setting the window to $1.32 \le x \le 1.34$ and $-0.001 \le y \le 0.001$ for a good view. You can also recognize the two zeroes from the table of values. Note that there's a sign change between 1.32 and 1.33 and another sign change between 1.33 and 1.34.

x	$f(x)$
1	2
1.32	0.0024
1.33	–0.00013
1.34	0.0017
2	14

a. The Rational Zeroes Theorem says that the only possible rational zeroes are of the form ±(a factor of 20)/(a factor of 3), so {±1, ±2, ±4, ±5, ±10, ±20, ±1/3, ±2/3, ±4/3, ±5/3, ±10/3, ±20/3}.

b. Checking the possibilities found in part **a**, the only rational zero is $x = 4/3 \approx 1.333$.

c. $f(1.32)$ is positive and $f(1.33)$ is negative, so by the Intermediate Value Theorem, f must have a zero between 1.32 and 1.33. There is no rational zero in the interval $1.32 < x < 1.33$. Therefore, the zero between 1.32 and 1.33 must be irrational.