

1. $\frac{1}{-6+8i} = \frac{1 \cdot (-6-8i)}{(-6+8i)(-6-8i)} = \frac{-6-8i}{36+48i-48i-64i^2} = \frac{-6-8i}{36+64} = \frac{-6-8i}{100} = -\frac{6}{100} - \frac{8}{100}i.$

2. a. The points will be reflections of each other across the x -axis.

b. $z \cdot \bar{z} = (a + bi) \cdot (a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$, which is a real number.

3. $x^3 = 1$

$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0 \quad [\text{you can get the quadratic by long division}]$$

Now use the quadratic formula to find the two zeroes of $x^2 + x + 1$.

Solutions: $\{1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i\}$

4. $P(x)$ must have the following zeroes: 0, i , and $-i$ (the last of these is due to the Complex Conjugates Theorem).

Therefore, $P(x)$ must have the following factors: x , $(x + i)$, and $(x - i)$. To make a 4th degree polynomial, we need a fourth linear factor, for which we can choose anything we wish.

Thus, one possible answer is:

$$P(x) = x \cdot x \cdot (x - i) \cdot (x + i)$$

$$P(x) = x^2 \cdot (x^2 + 1)$$

$$P(x) = x^4 + x^2$$

$$P(x) = 1x^4 + 0x^3 + 1x^2 + 0x + 0.$$

5. A complete response should include the following details:

- $Q(x)$ has 4 zeroes in the complex numbers (by the Fund. Thm. Of Algebra)
- $Q(x)$ has 2 non-real zeroes, which are $2 + i$ and $2 - i$ (the latter by the Cplx. Conj. Thm.)
- $Q(x)$ has 2 real zeroes. One of these real zeroes is 1. We know the only remaining unknown zero must also be real (it can't be non-real because non-real zeroes have to occur in conjugate pairs).

Note that the Rational Zeroes Theorem does not apply here because $Q(x)$ has real coefficients. (The Rational Zeroes Theorem requires integer coefficients.) So, any assertion about rational zeroes having the form $\pm(\text{factors of } 5)/(\text{factors of } 7)$ is incorrect.