Part A (25%)	_____
Part B (25%)	_____
Part C (25%)	_____
Part D (25%)	_____
overall	_____

General instructions: Write a complete, fully explained solution to each problem, unless otherwise indicated. The quality of your responses will be a factor in grading. You may assume any needed properties of exponents and logs, unless otherwise indicated.

Part A. Exponential and logarithmic functions

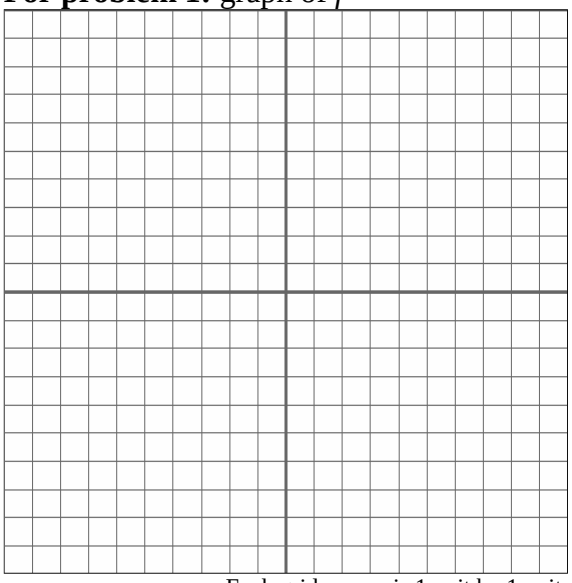
1. Let $f(x) = 2^x$. Find the inverse function $f^{-1}(x)$, state its domain and range, and draw its graph.

$f^{-1}(x)$ = _____

domain of f^{-1} = _____

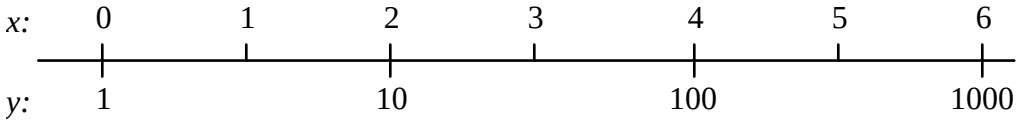
range of f^{-1} = _____

For problem 1: graph of f^{-1}



Each grid square is 1 unit by 1 unit.

2. A "ruler" is labeled with two numerical scales, as shown below:



What kind of ruler is this? Answer the following questions to explain how the numbers x on the top scale are related to the numbers y on the bottom scale.

a. Write an exponential equation relating x and y.

b. Write a logarithmic equation relating x and y.

3. Suppose that $F(x) = \log x$ and $G(x) = \ln x$. Describe the transformation that would transform the graph of $F(x)$ into the graph of $G(x)$. Justify your answer.

Part B. Properties of exponentials and logarithms

For all of the problems on this page, you may assume that a, b, c, M, and N are positive numbers and are not equal to 1. Your justifications may use any of the properties of exponents and logs that we have studied.

1. Prove or disprove this equation: $a^x = e^{x \ln a}$.

Circle one of these words to tell which you are doing: Proving Disproving

2. Prove or disprove this equation: $\log(M + N) = \log(M) + \log(N)$.

Circle one of these words to tell which you are doing: Proving Disproving

3. How are the quantities $\log_a b$ and $\log_b a$ related to each other? Justify your answer.

4. How are the quantities $\log_b c$ and $\log_b(1/c)$ related to each other? Justify your answer.

Part C. Working with exponentials and logarithms

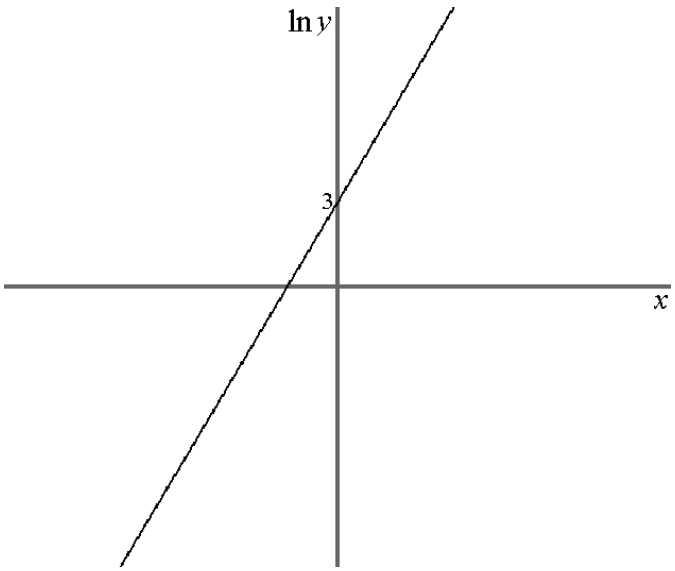
1. Given $w = \left(\frac{bx}{y^3}\right)^2$, write $\log_b w$ in terms of $\log_b x$ and $\log_b y$.

2. Solve these equations for x algebraically. Show your solving steps.

a. $\log_8(2x) = -2$.

b. $4 \log_3(2) = \log_3(4x)$.

3. Let y be a function of x. The given graph shows the relationship between x and $\ln y$.



What type of function is y as a function of x?

_____

Perform a calculation that justifies your answer.

Part D. Applications of exponentials and logarithms

1. a. An investment pays interest at an annual rate of 2.4% and is **compounded monthly**. Determine the number of years before the investment is worth 150% of its initial value. Show your work.

b. An investment pays interest at an annual rate of 2.5% and is **compounded continuously**. Determine the number of years before the investment is worth 150% of its initial value. Show your work.

2. This problem concerns the cooling of a warm object as time progresses. Let T represent the object's temperature (in degrees Celsius) as a function of time t (in minutes). Let T_m represent the constant temperature of the medium (i.e., room temperature).

Newton's Law of Cooling says that $\ln(T - T_m)$ will be linear. Suppose that for this object, the linear equation is:

$$\ln(T - T_m) = -0.1t + \ln(40)$$

a. Solve the given equation for T.

b. Suppose that the room temperature T_m is 20° Celsius. Find the temperature of the object at time $t = 0$ and at time $t = 5$.

At time $t = 0$: _____ At time $t = 5$: _____