Name ________________________________ Honors Pre-Calculus Test Answers

February 3, 2003 Sections 5.1–5.4

Part A. Proving identities

1. a. OQ = $\cos\alpha\cos\beta$
OP = $\cos(\alpha+\beta)$
PQ = $\sin\alpha\sin\beta$

b. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

2. $\cos(2u) = \cos(u + u)$
$= \cos u\cos u - \sin u\sin u$
$= \cos^2 u - \sin^2 u$
$= (1 - \sin^2 u) - \sin^2 u$
$= 1 - 2\sin^2 u$

3. a. Start with the identity from problem **2**, then substitute $x = 2u$ to get

$$\cos(x) = 1 - 2\sin^2(\tfrac{1}{2}x)$$
$$2\sin^2(\tfrac{1}{2}x) = 1 - \cos x$$
$$\sin^2(\tfrac{1}{2}x) = \tfrac{1-\cos x}{2}$$
$$\sin(\tfrac{1}{2}x) = \pm\sqrt{\tfrac{1-\cos x}{2}}$$

b. The sign depends on the quadrant that angle $\frac{1}{2}x$ falls in: + for the 1st or 2nd quadrant, – for the 3rd or 4th quadrant. You could use any 3rd or 4th quadrant angle as your numerical example.

4.

$$\frac{1+2\sin x\cos x}{(\cos x)^2}$$
$$= \frac{1}{(\cos x)^2} + \frac{2\sin x\cos x}{(\cos x)^2}$$
$$= \sec^2 x + 2\tan x$$
$$= (1+\tan^2 x) + 2\tan x$$
$$= 1 + 2\tan x + \tan^2 x$$
$$= (1+\tan x)^2$$

Part B. Using the identities

1. a.

$$\sin(\tfrac{11}{12}\pi) = \sin(\tfrac{2}{3}\pi + \tfrac{1}{4}\pi)$$
$$= \sin(\tfrac{2}{3}\pi)\cos(\tfrac{1}{4}\pi) + \cos(\tfrac{2}{3}\pi)\sin(\tfrac{1}{4}\pi)$$
$$= \tfrac{\sqrt{3}}{2}\cdot\tfrac{\sqrt{2}}{2} + \tfrac{-1}{2}\cdot\tfrac{\sqrt{2}}{2}$$
$$= \tfrac{\sqrt{6}-\sqrt{2}}{4}.$$

b. In the following, + sign is chosen because $\frac{11}{12}\pi$ is in 2nd quadrant.

$$\sin(\tfrac{11}{12}\pi) = \sin(\tfrac{1}{2}(\tfrac{11}{6}\pi))$$
$$= +\sqrt{\frac{1-\cos\frac{11}{6}\pi}{2}}$$
$$= +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$
$$= +\sqrt{\tfrac{1}{2} - \tfrac{\sqrt{3}}{4}}$$

2. a. $\sin(-x)\csc(\frac{\pi}{2} - x) = -\sin(x)\sec(x)$
$= -\sin(x)\,\frac{1}{\cos(x)} = -\tan(x)$.

b.

$$\frac{\csc^2 x}{(\csc^2 x)-2} = \frac{\csc^2 x}{(\csc^2 x)-2}\cdot\frac{\sin^2 x}{\sin^2 x}$$
$$= \frac{1}{1-2\sin^2 x} = \frac{1}{\cos(2x)} = \sec(2x).$$

3. a. $\tan(3x) + 2\sin(3x) = 0$
$\sin(3x)\,(\frac{1}{\cos(3x)} + 2) = 0$

$\sin(3x) = 0$ or $(\frac{1}{\cos(3x)} + 2) = 0$
$\sin(3x) = 0$ or $\cos(3x) = -\frac{1}{2}$

$3x = \pi n$ or $3x = \frac{2\pi}{3} + 2\pi n$
or $3x = \frac{4\pi}{3} + 2\pi n$

$x = \frac{\pi}{3}n$ or $x = \frac{2\pi}{9} + \frac{2\pi}{3}n$
or $x = \frac{4\pi}{9} + \frac{2\pi}{3}n$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}; \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9};$
$\frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}.$

b. Graph $Y_1 = \tan(3x)$ and $Y_2 = -2\sin(3x)$ using Xmin = 0, Xmax = 2π. Then circle the twelve x-values listed as answers to part **a**.