Name ________________________________ Honors Pre-Calculus Test

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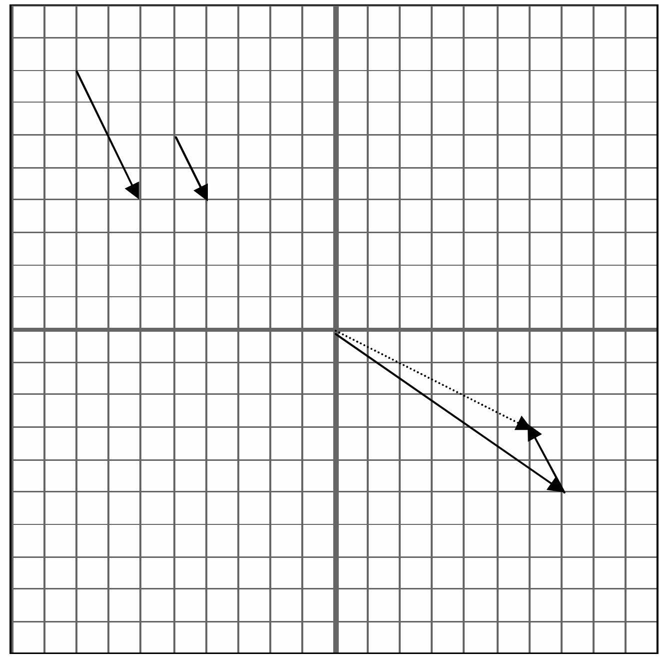
Part A. Vector concept and operations



1. a. $\frac{1}{2}\mathbf{v}$ goes in the same direction as **v**, but with half the length of **v**. See the pair of vectors in the upper part of the grid.

b. Here is one of a few possible approaches: $\mathbf{w} - \frac{1}{2}\mathbf{v} = \mathbf{w} + (-\frac{1}{2}\mathbf{v})$. Place vectors **w** and $(-\frac{1}{2}\mathbf{v})$ such that **w** ends where $(-\frac{1}{2}\mathbf{v})$ begins. Then draw the vector that completes the vector addition triangle, as shown by the dotted arrow on the grid.

c. $\mathbf{w} - \frac{1}{2}\mathbf{v} = \langle 7, -5\rangle - \frac{1}{2}\langle 2, -4\rangle = \langle 7, -5\rangle - \langle 1, -2\rangle = \langle 6, -3\rangle$.

2. a. It's $\mathbf{v} - \mathbf{u}$.

b. The vector from the origin to the midpoint of the opposite side of the triangle is $\mathbf{u} + \frac{1}{2}(\mathbf{v} - \mathbf{u}) = \frac{1}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$. This vector is a median of the triangle, and the centroid subdivides each median in a $\frac{2}{3} : \frac{1}{3}$ radio. So, $\mathbf{w} = \frac{2}{3}(\frac{1}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}) = \frac{1}{3}\mathbf{u} + \frac{1}{3}\mathbf{v}$.

Part B. Vector lengths, angles, dot products, and polar coordinates

1. a. $\mathbf{i} \cdot \mathbf{j} = \langle 1, 0\rangle \cdot \langle 0, 1\rangle = 1 \cdot 0 + 0 \cdot 1 = 0$.

b. $\mathbf{i} \cdot \mathbf{j} = 0$ because the vectors are perpendicular.

c. Prove that $a\mathbf{i} + b\mathbf{j} = a\langle 1, 0\rangle + b\langle 0, 1\rangle = \langle a, 0\rangle + \langle 0, b\rangle = \langle a, b\rangle$.

2. a. $\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle = u_1v_1 + u_2v_2 = v_1u_1 + v_2u_2 = \langle v_1, v_2\rangle \cdot \langle u_1, u_2\rangle$.

b. This statement is false. One possible counterexample is $\mathbf{v} = \mathbf{i}$ and $\mathbf{w} = \mathbf{j}$. Then $|\mathbf{v} \cdot \mathbf{w}| = |0| = 0$, but $|\mathbf{v}|\,|\mathbf{w}| = 1 \cdot 1 = 1$, so $|\mathbf{v} \cdot \mathbf{w}| \neq |\mathbf{v}|\,|\mathbf{w}|$.

3. a. One correct answer: ≈ (5, 126.87°) and (–5, 306.87°).
Any other correct answer would have the same radii, and angles coterminal to these.

b. $\langle -\frac{3}{5}, \frac{4}{5}\rangle$.

4. Let *O* represent the origin, let *P* represent the point whose polar coordinates are (2, 30°), and let *Q* represent the point whose polar coordinates are (–6, 330°).

a. Diagram should show a triangle with *O* at the origin, *P* in the 1st quadrant, and *Q* in the **2nd quadrant** at an angle of 150° (not in the 4th quadrant, due to the negative radius).

b. $\overrightarrow{OP} = \langle \sqrt{3}, 1\rangle$; $\overrightarrow{OQ} = \langle -3\sqrt{3}, 3\rangle$.

c. $\vec{OP} \cdot \vec{OQ} = \sqrt{3} \cdot (-3\sqrt{3}) + 1 \cdot 3 = -9 + 3 = -6.$
Another possibility would be to use the angle-between-two-vectors formula, which gives
$\vec{OP} \cdot \vec{OQ} = (\cos\theta)\,|\vec{OP}|\,|\vec{OQ}| = (-\frac{1}{2}) \cdot 2 \cdot 6 = -6.$

d. ***This part was omitted from the test for H block and any 55-minute makeups.***
Use Law of Cosines:
$PQ^2 = OP^2 + OQ^2 - 2(OP)(OQ)\cos\theta = 2^2 + 6^2 - 2\cdot2\cdot6\cdot(-\frac{1}{2}) = 40 + 12 = 52.$
So $PQ = \sqrt{52} \approx 7.21.$

Part C. Parametric equations

1. **a.** Direction vector is <–7, 6>.
One possible parametrization: $x = 3 - 7t$, $y = -1 + 6t$, where $0 \le t \le 1$.

b. One possible parametrization: $x = (5 \cos t) + 1$, $y = (5 \sin t) - 4$, where $\pi \le t \le 2\pi$.

2. If you chose **1a**: One possible method is to rewrite first equation as $t = (3 - x)/7$
and substitute into the second equation to get $y = -1 + 6((3 - x)/7 = (-6/7)x + 11/7$.

If you chose **1b**: Rewrite equations as $x - 1 = 5 \cos t$, $y + 4 = 5 \sin t$. Square both equations, then add to get $(x - 1)^2 + (y + 4)^2 = (5 \cos t)^2 + (5 \sin t)^2$. Simplify using Pythagorean identity to get $(x - 1)^2 + (y + 4)^2 = 25$.

3. **a.** <30 cos 65°, 30 sin 65°> ≈ <12.68, 27.19>.

b. Let $y = 0$ and solve the second equation for t using any method for solving quadratics. The only positive solution is $t \approx 1.897$, the time that the ball hits the ground. Then use first equation to find the horizontal distance: $x \approx (30 \cos 65°)\cdot 1.897 \approx 24.05$ ft.