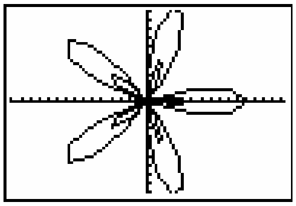
Name ________________________________ Honors Pre-Calculus Test

March 21, 2003 6.4–6.6

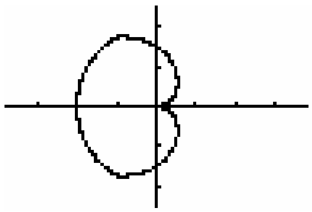
Part A. Polar coordinates and graphing

1. **a.**  **b.** $-5 < r < 11$.

c. longer leaves: 11; shorter leaves: 5.

2. **a.**

θ	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
r	0	1	2	2	2	1	0

b. 

3. **a.** some possibilities: $r = \cos\theta$; $r = 2$.

b. some possibilities: $r = \sec\theta$; $\theta = 2$.

c. some possibilities: $r = \theta$; $r = e^{\theta}$.

Part B. Complex numbers in polar form

1. **a.** $zw = (2-2i)(-\frac{1}{2}+\frac{\sqrt{3}}{2}i) = -1 + i + \sqrt{3}\,i - \sqrt{3}\,i^2 = (-1+\sqrt{3}) + (1+\sqrt{3})i$.

b. $zw = (2\sqrt{2}\text{ cis}(-\frac{\pi}{4}))\ (1 \text{ cis } \frac{2\pi}{3}) = 2\sqrt{2}\text{ cis }(\frac{5\pi}{12})$.

c. Check that $2\sqrt{2}\cos(\frac{5\pi}{12}) = -1+\sqrt{3}$ and $2\sqrt{2}\sin(\frac{5\pi}{12}) = 1+\sqrt{3}$.

2. **a.** $z^{10} = (2 \text{ cis } \frac{\pi}{6})^{10} = 2^{10}\text{ cis }\frac{10\pi}{6}$.

b. $\frac{1}{z} = (1 \text{ cis } 0)/(2 \text{ cis } \frac{\pi}{6}) = \frac{1}{2}\text{ cis}(-\frac{\pi}{6})$.

3. In polar form, $-125 = 125 \text{ cis } \pi$. Its cube roots are $125^{1/3}\text{ cis}(\frac{\pi+2\pi k}{3})$ for $k = 0, 1, 2$.
So, the cube roots are $5 \text{ cis}\frac{\pi}{3}$, $5 \text{ cis } \pi$, and $5 \text{ cis}\frac{5\pi}{3}$. Diagram should show three points on the circle of radius 5, evenly spaced at angles of $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$.

Part C. Relating complex numbers, polar coordinates, and vectors

a. $iz = i(a + bi) = ai + bi^2 = -b + ai$.

b. $iz = (1 \text{ cis } \frac{\pi}{2})(r \text{ cis } \theta) = r \text{ cis } (\frac{\pi}{2} + \theta)$.

c. any diagram showing z and iz having the same radius r, and with iz having an angle that is $\frac{\pi}{2}$ more than the angle of z

d. $\mathbf{u} = \langle a, b\rangle$; $\mathbf{v} = \langle -b, a\rangle$.

e. $\mathbf{u}\cdot\mathbf{v} = a(-b) + ba = 0$.

f. **u** and **v** are perpendicular because $\mathbf{u}\cdot\mathbf{v} = 0$, or because their angles differ by $\frac{\pi}{2}$,
or because they have opposite reciprocal slopes ($\frac{b}{a}$ and $-\frac{a}{b}$).