Name ______________________________ Honors Pre-Calculus Test
April 11, 2003 Chapter 7

Part A. Matrix operations

1. $\begin{bmatrix} 8 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0.25 & 0.30 & 0.20 & 0.25 \\ 0.25 & 0.00 & 0.35 & 0.40 \end{bmatrix} = \begin{bmatrix} 3.5 & 2.4 & 3.7 & 4.4 \end{bmatrix}$

OR $\begin{bmatrix} 0.25 & 0.25 \\ 0.30 & 0.00 \\ 0.20 & 0.35 \\ 0.25 & 0.40 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 6 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 2.4 \\ 3.7 \\ 4.4 \end{bmatrix}$

2. a. (If your drawing was incorrect, I corrected it on your test.)

b. The largest entry is a 3 in the "from A to D" location. It tells that there are 3 two-step paths of communication from A to D.

c. It gives the number of paths of communication between each pair of points, involving either one or two steps (i.e., paths of length two or less).

Part B. Linear systems

1. Here is one of many possible sequence of row operations:
$2R_2 + R_3 \rightarrow R_3$
$\frac{1}{2} R_3 \rightarrow R_3$
$R_2 - 3R_3 \rightarrow R_2$
$R_1 + 3R_3 \rightarrow R_1$
$R_1 - 2R_2 \rightarrow R_1$

From this or any other sequence of row operations into rref, the solution should be (5, –1, 3).

2. $\frac{x+7}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$

$x + 7 = A(x - 2) + B(x + 3)$
$1x + 7 = (A + B)x + (-2A + 3B)$

Need to solve: $A + B = 1$, $-2A + 3B = 7$.

$$\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 1.8 \end{bmatrix}$$

Answer: $\frac{x+7}{x^2+x-6} = \frac{-0.8}{x+3} + \frac{1.8}{x-2}$

3. a. $(z, 5, 3 - 2z, z)$.

b. No. When a system does not have a unique solution, the coefficient matrix does not have an inverse matrix.

Part C. Linear transformations

1. a. $x' = 2x - 3y$, $y' = -4x + 5y$.

b. *Solving the transformation matrix equation using an inverse:*

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -3 & 17 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} -3 & 17 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} -18 & -11 \end{bmatrix}$$

Setting up a linear system and solving using rref form:

$2a - 3b = -3$
$-4a + 5b = 17$

$$\left[\begin{array}{cc|c} 2 & -3 & -3 \\ -4 & 5 & 17 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & -18 \\ 0 & 1 & -11 \end{array}\right]$$

$(a, b) = (-18, -11)$

c. The determinant is $2 \cdot 5 - (-4)(-3) = -2$.
The negative determinant tells that the transformation is orientation reversing.
The absolute value of 2 tells that the transformation doubles the area of any region.

2. a. $$\begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 - \frac{3\sqrt{3}}{2} & \sqrt{3} - \frac{3}{2} \end{bmatrix} \approx \begin{bmatrix} -3.598 & 0.232 \end{bmatrix}$$

b. We need an angle α such that $\cos\alpha = -\frac{1}{2}$ and $\sin\alpha = \frac{\sqrt{3}}{2}$.
So α may be $2\pi/3$ radians (=120°), or any coterminal angle.

3. a.

$$\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} \cos(-\beta) & \sin(-\beta) \\ -\sin(-\beta) & \cos(-\beta) \end{bmatrix} =$$

$$\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} =$$

$$\begin{bmatrix} \cos^2\beta + \sin^2\beta & -\cos\beta\sin\beta + \sin\beta\cos\beta \\ -\sin\beta\cos\beta + \cos\beta\sin\beta & \cos^2\beta + \sin^2\beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b. The "1,0; 0,1" matrix tells us that the composite transformation is simply $x' = x$, $y' = y$; that is, a transformation that leaves every point in its original position. This makes sense because a rotation by angle β followed by a rotation by the opposite angle $-\beta$ would return every point to its original position.