Name ______________________________ Honors Pre-Calculus Test
May 23, 2003 9.1–9.3 plus extras

## Part A. Transition matrices

**a.** $T = \begin{bmatrix} .90 & .03 & .07 \\ .02 & .92 & .06 \\ .12 & .09 & .79 \end{bmatrix}$

**b.** $[0\ \ 100\ \ 0] \cdot T^4 \approx [9\ \ 75\ \ 16]$

**c.** 36.3%, 40.1%, 23.6% (easiest way to get these is to look at a large power of *T*)

## Part B. Counting

**1.** $\dfrac{{}_nP_r}{{}_nC_r} = \dfrac{\frac{n!}{(n-r)!}}{\frac{n!}{(n-r)!\,r!}} = r!$

**2.** **a.** ${}_{15}C_3 = 455.$

**b.** $2^{15} = 32768.$

**3.** **a.** ${}_{51}P_2 = 2550.$

**b.** (total no. of committees) – (committees with no D's) – (committees with no R's) =
${}_{100}C_7 \quad - \quad {}_{52}C_7 \quad - \quad {}_{49}C_7 \quad \approx 1.579 \cdot 10^{10}.$

**4.** The relevant term of the expansion is ${}_9C_3\,(x^2)^3\,(\tfrac{3}{x})^6 = 84\,x^6\,729\,x^{-6} = 61236.$

## Part C. Probability

**1.** $\dfrac{{}_{26}P_6}{26^6} \approx 0.537.$

**2.** Making a tree diagram helps here.
$P$(pair with different colors) = $P(GB) + P(BG) = \frac{8}{13} \cdot \frac{5}{12} + \frac{5}{13} \cdot \frac{13}{12} = \frac{80}{156} \approx 0.513.$

**3.** **a.** Making a tree diagram helps here.
$P(R) = P(A \text{ and } R) + P(B \text{ and } R)$
$= P(A)\,P(R \mid A) + P(B)\,P(R \mid B)$
$= \frac{1}{2} \cdot \frac{4}{6} + \frac{1}{2} \cdot \frac{2}{5} = \frac{8}{15} \approx 0.533.$

**b.** $P(B \mid R) = P(B \text{ and } R) / P(R) = (\frac{1}{5}) / (\frac{8}{15}) = \frac{3}{8}.$

**4.** **a.** $\dfrac{{}_{13}C_3 \cdot {}_{39}C_2}{{}_{52}C_5} \approx 0.082.$

**b.** (no. of hands with ≥3 ♦'s) = (total no. of hands) – (hands with 5 ♦'s) – (hands with 4 ♦'s)
$= {}_{52}C_5 \quad - \quad {}_{13}C_5 \quad - \quad {}_{13}C_4\,{}_{39}C_1.$

Then, $P$(no. of hands with ≥3 ♦'s) = $\dfrac{{}_{52}C_5 - {}_{13}C_5 - {}_{13}C_4\,{}_{39}C_1}{{}_{52}C_5} \approx 0.989.$

**5.** **a.** $P(X \text{ and } Y) = 0$ because the events are mutually exclusive. But $P(X) = \frac{1}{2}$ and $P(Y) = \frac{1}{6}$.
Thus $P(X \text{ and } Y) \neq P(X)P(Y)$, so the events are **not independent**.

**b.** The game favors Amy with an expected gain of $\frac{1}{2}(\$2) \cdot \frac{1}{6}(\$5) \cdot \frac{1}{3}(\$ -5) \approx \$0.167.$