

Equivalence Relationships

An equivalence relationship, which we will denote by a diamond symbol \diamond for the general case, is any relationship that has the following 3 properties:

1. $A \diamond A$ That is to say a thing is equivalent to itself. This is referred to as the **Reflexive property**.
2. $A \diamond B \Rightarrow B \diamond A$ That is to say if a thing is equivalent to another thing, then that other thing is equivalent to it. This is referred to as the **Symmetric property**.
3. $A \diamond B$ and $B \diamond C \Rightarrow A \diamond C$ That is to say if a thing is equivalent to another thing and that thing is equivalent to a third, the first and third thing are equivalent. This is known as the **Transitive Property**.

Questions:

1. What are some equivalence relationships you have experienced in your study of mathematics?
2. Can you think of any relationships outside of math that are equivalence relationships?
3. Can you name some relationships that are not equivalence relationships because they lack some of the properties?

Using Equivalence

Once we understand how equivalence relationships behave we need to learn how to use them. In algebra we make great use of equality “=”, in geometry we will use similarity, congruence, equality, identity and logical equivalence “ $\sim, \cong, =, \equiv, \Leftrightarrow$ ”. In time we will learn how to use and prove all of these relationships. The last example, logical equivalence, is used the most. This is because all definitions in geometry are logical equivalences. For example, M is the midpoint of $\overline{AB} \Leftrightarrow \overline{AM} \cong \overline{MB}$ and M is on \overline{AB} (or A, M, B are collinear). If you are trying to prove M is the midpoint of \overline{AB} you can do this by showing $\overline{AM} \cong \overline{MB}$ and claiming M is the midpoint of \overline{AB} by the definition of midpoint. You can also prove the former by showing the latter, this is how definition is used.

Proof

A proof is a chain of statements leading from a set of given facts to a conclusion. Most proofs are in fact implication (If...Then) statements where the “if” part is(are) some given fact(s) and the “then” part is what you are trying to prove. In the early part of this course we will be very careful to explain the reason behind every statement. The form of these proofs is a tradition in mathematics known as the “two column proof” where one column contains a statement that is advancing the given facts toward the conclusion and the other details the reason we know the statement is true.

ENOUGH OF THIS BLATHER! PLEASE SHOW US AN EXAMPLE!!

Yeah, ok...

$AB = 6,$
Given : $BC = 6,$ Show: B is the midpoint of \overline{AC}
 A, B, C are collinear

Statement	Reason
1. $AB = 6$ (It seems tedious but you have to introduce the given information to the proof)	Given
2. $BC = 6$	Given
3. $\overline{AB} \cong \overline{BC}$ (The reason comes from line 1 and 2 above which form the definition of congruence)	1,2, Definition of \cong
4. A, B, C are collinear (this will often be inferred from a diagram)	Given
5. B is the midpoint of \overline{AC}	3,4, Definition of midpoint.

For Homework please read section 1-4 in your books and do 1-7 on page 26-27. And attach it to this sheet.